## GATE-2013

## ME:MECHANICALENGINEERING

## Read the following instructions carefully.

1. All questions in this paper are of objective type.
2. There are a total of 65 questions carrying 100 marks.
3. Questions 1 through 25 are 1-mark questions, question 26 through 55 are 2-mark questions.
4. Questions 48 to 51 (2 pairs) common data questions and question pairs (Q. 52 and Q .53 ) and (Q. 54 and Q .55 ) are linked answer questions. The answer to the second question of the above pair depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is un-attempted, then the answer to the second question in the pair will not be evaluated.
5. Questions 56-65 belong to general aptitude (GA). Questions 56-60 will carry 1-mark each, and questions $61-65$ will carry 2 -marks each. The GA questions will begin on a fresh page.
6. Un-attempted questions will carry zero marks.
7. Wrong answers will carry NEGATIVE marks. F or Q. 1 to Q .25 and Q .56 - $\mathrm{Q} .60,1 / 3$ mark will be deducted for each wrong answer. F or Q. 26 to Q. 51, and Q.61-Q.65, 2/3 mark will be deducted for each wrong answer. The question pairs (Q.52, Q.53) and (Q. 54, Q. 55) are questions with linked answers. There will be negative marks only for wrong answer to the first question of the linked answer question pair i.e. for Q. 52 and Q .54 , 2/3 mark will be deducted for each wrong answer. There is no negative marking for Q. 53 and Q.55..
Q. 1 to Q .25 carry one mark each.
8. The partial differential equation

$$
\frac{\partial u}{\partial t}+u \frac{\partial u}{\partial \lambda}=\frac{\partial^{2} u}{\partial x^{2}} \text { is } a
$$

(a) linear equation of order 2
(b) non-linear equation of order 1
(c) linear equation of order 1
(d) non-linear equation of order 2
2. The eigenvalues of a symmetric matrix are all
(a) complex with non-zero positive imaginary part
(b) complex with non-zero negative imaginary part
(c) real
(d) pureimaginary
3. $M$ atch the CORRECT pairs

| Numerical Integration <br> Scheme | Order of Fitting <br> Polynomial |
| :--- | :--- |
| P Simpson's 3/8 Rule | 1 First |
| Q Trapezoidal Rule | 2 Second |
| R Simpson's 1/3 Rule | 3 Third |

(a) $\mathrm{P}-2, \mathrm{Q}-1, \mathrm{R}-3$
(b) $\mathrm{P}-3, \mathrm{Q}-2, \mathrm{R}-1$
(c) $\mathrm{P}-1, \mathrm{Q}-2, \mathrm{R}-3$
(d) $\mathrm{P}-3, \mathrm{Q}-1, \mathrm{R}-2$
4. A rod of length $L$ having uniform cross-sectional area A is subjected to a tensile force P as shown in the figure below If the Young's modulus of the material varies linearly from $E_{1}$ to $E_{2}$ along the length of the rod the normal stress developed at the section-SS is

(c) $\frac{P E_{2}}{A E_{1}}$
(d) $\frac{P E_{1}}{A E_{2}}$
5. Two threaded bolts A and B of same material and length are subjected to identical tensile load. If the elastic strain energy stored in bolt A is 4 times that of bolt $B$ and the mean diameter of bolt $A$ is 12 mm , the mean diameter of bolt $B$ in mm is
(a) 16
(b) 24
(c) 36
(d) 48
6. A link $O B$ is rotating with a constant angular vel ocity of $2 \mathrm{rad} / \mathrm{s}$ in counter clockwise direction and a block is sliding radially outward on it with an uniform velocity of $0.75 \mathrm{~m} / \mathrm{s}$ with respect to the rod, as shown in the figure below. If $\mathrm{OA}=1 \mathrm{~m}$ the magnitude of the absolute acceleration of the block at location $A$ in $\mathrm{m} / \mathrm{s}^{2}$ is

(a) 3
(b) 4
(c) 5
(d) 6
7. For steady, fully developed flow inside a straight pipe of diameter D neglecting gravity effects the pressure drop $\Delta p$ over a length $L$ and the wall shear stress $\tau_{\text {w }}$ are related by
(a) $\tau_{w}=\frac{\Delta p D}{4 L}$
(b) $\tau_{w}=\frac{\Delta p D^{2}}{4 L^{2}}$
(c) $\tau_{\mathrm{w}}=\frac{\Delta \mathrm{pD}}{2 \mathrm{~L}}$
(d) $\tau_{w}=\frac{4 \Delta p L}{D}$
8. The pressure, dry bulb temperature and relative humidity of air in a room are 1 bar, $30^{\circ} \mathrm{C}$ and $70 \%$, respectively. If the saturated steam pressure at $30^{\circ} \mathrm{C}$ is 4.25 kPa , the specific humidity of the room air in kg water vapour/kg dry air is
(a) 0.0083
(b) 0.0101
(c) 0.0191
(d) 0.0232
9. Consider one-dimensional steady state heat conduction, without heat generation, in a plane wall with boundary conditions as shown in the figure below. The conductivity of the wall is given by $k=k_{0}+b T$, where $k_{0}$ and $b$ are positive constants, and $T$ is temperature


As $x$ increases, the temperature gradient ( $\mathrm{dT} / \mathrm{dx}$ ) will
(a) remain constant
(b) be zero
(c) increase
(d) decrease
10. In a rolling process, the state of stress of the material undergoing deformation is
(a) pure compression
(b) pure shear
(c) compression and shear
(d) tension and shear
11. Match the CORRECT pairs

| Processes | Characteristics/ <br> Applications |
| :--- | :--- |
| P Friction Welding | 1. Non-consumable <br> electrode |
| Q Gas Metal Arc Welding | 2. J oining of thick plates |
| R Tungsten Inert <br> Gas Welding | 3. Consumable electrode <br> wire |
| S Electroslag Welding | 4. J oining of cylindrical <br> dissimilar materials |

(a) P-4, Q-3, R-1, S-2
(b) P-4, Q-2, R-3, S-1
(c) $\mathrm{P}-2, \mathrm{Q}-3, \mathrm{R}-4, \mathrm{~S}-1$
(d) P-2, Q-4, R-1, S-3
12. A metric thread of pitch 2 mm and thread angle $60^{\circ}$ is inspected for its pitch diameter using 3-wire method. The diameter of the best size wire in mm is
(a) 0.866
(b) 1.000
(c) 1.154
(d) 2.000
13. Customers arrive at a ticket counter at a rate of 50 per hr and tickets are issued in the order of their arrival. The average time taken for issuing a ticket is 1 min . A ssuming that customer arrivals form a Poisson process and service times are exponentially distributed, the average waiting time in queue in min is
(a) 3
(b) 4
(c) 5
(d) 6
14. In simple exponential smoothing forecasting, to give higher weightage to recent demand information, the smoothing constant must be close to
(a) -1
(b) zero
(c) 0.5
(d) 1.0
15. A steel bar 200 mm in diameter is turned at a feed of $0.25 \mathrm{~mm} / \mathrm{rev}$ with a depth of cut of 4 mm . The rotational speed of the workpiece is 160 rpm . The material removal rate in $\mathrm{mm}^{3} / \mathrm{s}$ is
(a) 160
(b) 167.6
(c) 1600
(d) 1675.5
16. A cube shaped casting solidifies in 5 mm . The solidification time in min for a cube of the same material, which is 8 times heavier than the original casting, will be
(a) 10
(b) 20
(c) 24
(d) 40
17. For a ductile material, toughness is a measure of
(a) resistance to scratching
(b) ability to absorb energy up to fracture
(c) ability to absorb energy till elastic limit
(d) resistance to indentation
18. In order to have maximum power from a Pelton turbine, the bucket speed must be
(a) equal to the jet speed
(b) equal to half of the jet speed
(c) equal to twice the jet speed
(d) independent of the jet speed
19. Consider one-dimensional steady state heat conduction along $x$-axis ( $0 \leq x \leq L$ ), through a plane wall with the boundary surfaces ( $x=0$ and $x=L$ ) maintained at temperatures of $0^{\circ} \mathrm{C}$ and $100^{\circ} \mathrm{C}$. Heat is generated uniformly throughout the wall. Choose the CORRECT statement
(a) The direction of heat transfer will be from the surface at $100^{\circ} \mathrm{C}$ to the surface at $0^{\circ} \mathrm{C}$
(b) The maximum temperature inside the wall must be greater than $100^{\circ} \mathrm{C}$
(c) The temperature distribution is linear within the wall
(d) The temperature distribution is symmetric about the mid-plane of the wall
20. A cylinder contains $5 \mathrm{~m}^{3}$ of an ideal gas at a pressure of 1 bar. This gas is compressed in a reversible isothermal process till its pressure increases to 5 bar The work in kJ required for this process is
(a) 804.7
(b) 953.2
(c) 981.7
(d) 1012.2
21. A long thin walled cylindrical shell, closed at both the ends, is subjected to an internal pressure. The ratio of the hoop stress (circumferential stress) to longitudinal stress developed in the shell is
(a) 0.5
(b) 1.0
(c) 2.0
(d) 4.0
22. If two nodes are observed at a frequency of 1800 rpm during whirling of a simply supported long slender rotating shaft, the first critical speed of the shaft in rpm is
(a) 200
(b) 450
(c) 600
(d) 900
23. A planar dosed kinematic chain is formed with rigid links $P Q=2.0 \mathrm{~m}, \mathrm{QR}=3.0 \mathrm{~m}, \mathrm{RS}=2.5 \mathrm{~m}$ and $\mathrm{SP}=2.7 \mathrm{~m}$ with all revol ute joints. The link to befixed toobtain a double rocker (rocker-rocker) mechanism is
(a) PQ
(b) $Q R$
(c) RS
(d) SP
24. Let $x$ be a normal random variable with mean 1 and variance 4 . The probability $\mathrm{P}\{\mathrm{X}<0\}$ is
(a) 0.5
(b) greater than zero and less than 0.5
(c) greater than 0.5 and less than 1.0
(d) 1.0
25. Choose the CORRECT set of functions, which are linearly dependent
(a) $\sin x, \sin ^{2} x$ and $\cos ^{2} x$
(b) $\cos x, \sin x$ and $\tan x$
(c) $\cos 2 x, \sin ^{2} x$ and $\cos ^{2} x$
(d) $\cos 2 x, \sin x$ and $\cos x$
Q. 26 to Q. 55 carry two marks each.
26. The following surface integral is to be evaluated over a sphere for the given steady velocity vector field $\mathrm{F}=\mathrm{xi}+\mathrm{yj}+\mathrm{zk}$ defined with respect to a Cartesian coordinate system having, $i, j$ and $k$ as unit base vectors

$$
\iint_{\mathrm{s}} \frac{1}{4}(\mathrm{~F} . \mathrm{n}) \mathrm{dA}
$$

where $S$ is the sphere, $x^{2}+y^{2}+z^{2}=1$ and $n$ is the outward unit normal vector to the sphere. Thevalue of the surface integral is
(a) $\pi$
(b) $2 \pi$
(c) $3 \pi / 4$
(d) $4 \pi$
27. The function $f(t)$ satisfies the differential equation $\frac{d^{2} f}{d t^{2}}+f=0$ and the auxiliary conditions, $f(0)=0$, $\frac{d f}{d t}(0)=4$. The $L$ aplace transform of $f(t)$ is given by
(a) $\frac{2}{s+1}$
(b) $\frac{4}{s+1}$
(c) $\frac{4}{s^{2}+1}$
(d) $\frac{2}{s^{4}+1}$
28. Specific enthalpy and velocity of steam at inlet and exit of a steam turbine, running under steady state, are as given below
$\begin{array}{lcc} & \begin{array}{c}\text { Specific enthalpy } \\ (\mathrm{kJ} / \mathrm{kg})\end{array} & \begin{array}{c}\text { Velocity } \\ (\mathrm{m} / \mathrm{s})\end{array} \\ \text { Inlet steam condition } & 3250 & 180 \\ \text { Exit steam condition } & 2360 & 5\end{array}$ The rate of heat loss from the turbine per kg of steam flow rate is 5 kW N eglecting changes potential energy of steam, the power developed in kW by the steam turbine per kg of steam flow rate, is
(a) 901.2
(b) 911.2
(c) 17072.5
(d) 17082.5
29. Water is coming out from a tap and falls vertically downwards. At the tap opening, the stream diameter is 20 mm with uniform velocity of $2 \mathrm{~m} / \mathrm{s}$. Acceleration due to gravity is $9.81 \mathrm{~m} / \mathrm{s}^{2}$. Assuming steady, in viscid flow, constant atmospheric pressure everywhere and neglecting curvature and surface tension effects, the diameter in mm of the stream 0.5 m . below the tap is approximately.
(a) 10
(b) 15
(c) 20
(d) 25
30. A steel ball of diameter 60 mm is initially in thermal equilibrium at $1030^{\circ} \mathrm{C}$ in a furnace. It is suddenly removed from the furnace and cooled in ambient air at $30^{\circ} \mathrm{C}$, with convective heat transfer coefficient $=$ $20 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$. The thermo-physical properties of steel are density $\rho=7800 \mathrm{~kg} / \mathrm{m}^{3}$, conductivity $\mathrm{k}=40 \mathrm{~W} / \mathrm{mK}$ and specific heat $\mathrm{c}=600 \mathrm{~J} / \mathrm{kg} / \mathrm{K}$. The time required in seconds to cool the steel ball in air from $1030^{\circ} \mathrm{C}$ to $430^{\circ} \mathrm{C}$ is
(a) 519
(b) 931
(c) 1195
(d) 2144
31. A flywheel connected to a punching machine has to supply energy of 400 Nm while running at a mean angular speed of $20 \mathrm{rad} / \mathrm{s}$. If the total fluctuation of speed is not to exceed $\pm 2 \%$, the mass moment of inertia of the flywheel in $\mathrm{kg} / \mathrm{m}^{2}$ is
(a) 25
(b) 50
(c) 100
(d) 125
32. A compound gear train with gears $P, Q, R$ and $S$ has number of teeth 20, 40, 15 and 20, respectively. Gears $Q$ and $R$ are mounted on the same shaft as shown in the figure below. The diameter of the gear $Q$ is twice that of the gear R. If the module of the gear $R$ is 2 mm , the center distance in mm between gears $P$ and $S$ is

33. A pin J ointed uniform rigid rod of weight $W$ and length L is supported horizontally by an external force F as shown in the figure below. The force $F$ is suddenly removed. At the instant of force removal, the magnitude of vertical reaction developed at the support is

(a) zero
(b) $W / 4$
(c) $W / 2$
(d) W
34. Two cutting tools are being compared for a machining operation. The tool life equations are
Carbide tool $\mathrm{VT}^{1.6}=3000$
HSS tool $\mathrm{VT}^{0.6}=200$
where V is the cutting speed in $\mathrm{m} / \mathrm{min}$ and T is the tool life in min. The carbide tool will provide higher tool life if the cutting speed in $\mathrm{m} / \mathrm{min}$ exceeds
(a) 15.0
(b) 39.4
(c) 49.3
(d) 60.0
35. In a CAD package, mirror image of a 2 D point $\mathrm{P}(5.10)$ is to be obtained about a line which passes through the origin and makes an angle of $45^{\circ}$ counterclockwise with the X-axis. The coordinates of the transformed point will be
(a) $(7.5,5)$
(b) $(10,5)$
(c) $(7.5,-5)$
(d) $(10,-5)$
36. A linear programming problem is shown below

Maximize $3 x+7 y$
Subject to

$$
\begin{aligned}
3 x+7 y & \leq 10 \\
4 x+6 y & \leq 8 \\
x, y & \geq 0
\end{aligned}
$$

It has
(a) an unbounded objective function
(b) exactly one optimal solution
(c) exactly two optimal solutions
(d) infinitely many optimal solutions
37. Cylindrical pins of $25^{+0.020} \mathrm{~mm}$ diameter are electroplated in a shop. Thickness of the plating is $30 \pm 20$ micron. N eglecting gage tol erances, the size of the GO gage in mm to inspect the plated components is
(a) 25.042
(b) 25.052
(c) 25.074
(d) 25.084
38. During the electrochemical machining (ECM) of iron (atomic weight $=56$, valency $=2$ ) at current of 1000 A with $90 \%$ current efficiency, the material removal rate was observed to be $0.26 \mathrm{gm} / \mathrm{s}$. If Titanium (atomic weight $=48$, valency $=3$ ) is machined by the ECM process at the current of 2000 A with $90 \%$ current efficiency, the expected material removal rate in gm/s will be
(a) 0.11
(b) 0.23
(c) 0.30
(d) 0.52
39. A single degree of freedom system having mass 1 kg and stiffness $10 \mathrm{kN} / \mathrm{m}$ initially at rest is subjected to an impulse force of magnitude 5 kN for $10^{-4}$ seconds. The amplitude in mm of the resulting free vibration is
(a) 0.5
(b) 1.0
(c) 5.0
(d) 10.0
40. A bar is subjected to fluctuating tensile load from 20 kN to 100 kN . The material has yield strength of 240 M Pa and endurance limit in reversed bending is 160 MPa . According to the Soderberg principle, the area of crosssection in $\mathrm{mm}^{2}$ of the bar for a factor of safety of 2 is
(a) 400
(b) 600
(c) 750
(d) 1000
41. A simply supported beam of length $L$ is subjected to a varying distributed load $\sin (3 \pi \mathrm{x} / \mathrm{L}) \mathrm{Nm}^{-1}$, where the distance $x$ is measured from the left support. The magnitude of the vertical reaction force in $N$ at the left support is
(a) zero
(b) $\mathrm{L} / 3 \pi$
(c) $\mathrm{L} / \pi$
(d) $2 \mathrm{~L} / \pi$
42. Two large diffuse gray parallel plates, separated by a small distance, have surfacetemperatures of 400 K and 300 K . If the emissivities of the surfaces are 0.8 and the StefanBoltzmann constant is $5.67 \times 10^{-8} \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}^{4}$, the net radiation heat exchange rate in $\mathrm{kW} / \mathrm{m}^{2}$ between the two plates is
(a) 0.66
(b) 0.79
(c) 0.99
(d) 3.96
43. A hinged gate of length 5 m , inclined at $30^{\circ}$ with the horizontal and with water mass on its left, is shown in the figure below. Density of water is $1000 \mathrm{~kg} / \mathrm{m}^{3}$. The minimum mass of the gate in kg per unit width (perpendicular to the plane of paper) required to keep it closed is

(a) 5000
(b) 6600
(c) 7546
(d) 9623
44. The pressure, temperature and velocity of air flowing in a pipe are 5 bar, 500 K and $50 \mathrm{~m} / \mathrm{s}$, respectively. The specific heats of air at constant pressure and at constant volume are $1.005 \mathrm{~kJ} / \mathrm{kgK}$ and $0.718 \mathrm{~kJ} / \mathrm{kgK}$, respectively. Neglect potential energy. If the pressure and temperature of the surroundings are 1 bar and 300 K respectively the available energy in $\mathrm{kJ} / \mathrm{kg}$ of the air stream is
(a) 170
(b) 187
(c) 191
(d) 213
45. The probability that a student knows the correct answer to a multiple choice question is $\frac{2}{3}$. If the student does not know the answer, then the student guesses the answer. The probability of the guessed answer being correct is $\frac{1}{4}$. Given that the student has answered the question correctly, the conditional probability that the student knows the correct answer is
(a) $\frac{2}{3}$
(b) $\frac{3}{4}$
(c) $\frac{5}{6}$
(d) $\frac{8}{9}$
46. The solution to the differential equation $\frac{d^{2} u}{d x^{2}}-k \frac{d u}{d x}=0$ where $k$ is a constant subjected to the boundary conditions $u(0)=0$ and $u(L)=U$ is
(a) $u=U \frac{X}{L}$
(b) $u=U\left(\frac{1-e^{k x}}{1-e^{k L}}\right)$
(c) $u=U\left(\frac{1-e^{-k x}}{1-e^{-k L}}\right)$
(d) $u=U\left(\frac{1+e^{k x}}{1+e^{k L}}\right)$
47. The value of the definite integral $\int_{1}^{e} \sqrt{x} \ln (x) d x$ is
(a) $\frac{4}{9} \sqrt{\mathrm{e}^{3}}+\frac{2}{9}$
(b) $\frac{2}{9} \sqrt{\mathrm{e}^{3}}-\frac{4}{9}$
(c) $\frac{2}{9} \sqrt{\mathrm{e}^{3}}+\frac{4}{9}$
(d) $\frac{4}{9} \sqrt{\mathrm{e}^{3}}-\frac{2}{9}$

## COMMON DATA QUESTIONS

Common Data for Questions 48 and 49:
A single riveted lap joint of two similar plates as shown in the figure below has the following geometrical and material details

width of the plate $w=200 \mathrm{~mm}$, thickness of the plate $\mathrm{t}=5 \mathrm{~mm}$, number of rivets $\mathrm{n}=3$, diameter of the rivet $d_{r}=10 \mathrm{~mm}$, diameter of the rivet hole $d_{h}=11 \mathrm{~mm}$, allowable tensile stress of the plate $\sigma_{p}=200 \mathrm{MPa}$, allowable shear stress of the rivet $\sigma_{s}=100 \mathrm{MPa}$ and allowable bearing stress of the rivet $\sigma_{c}=150 \mathrm{MPa}$.
48. If the rivets are to be designed to avoid crushing failure, the maximum permissible load $P$ in kN is
(a) 7.50
(b) 15.00
(c) 22.50
(d) 30.00
49. If the plates are to be designed to avoid tearing failure, the maximum permissible load $P$ in $k N$ is
(a) 83
(b) 125
(c) 167
(d) 501

Common Data for Questions 50 and 51:
Water (specific heat, $c_{p}=4.18 \mathrm{~kJ} / \mathrm{kgK}$ ) enters a . pipe at a rate of $0.01 \mathrm{~kg} / \mathrm{s}$ and a temperature of $20^{\circ} \mathrm{C}$. The pipe, of diameter 50 mm and length 3 m , is subjected to a wall heat flux $q_{w}^{\prime \prime}$ in $W / m^{2}$.
50. If $q_{w}^{\prime \prime}=2500 x$, where $x$ is in $m$ and in the direction of flow ( $x=0$ at the inlet), the bulk mean temperature of the water leaving the pipe in ${ }^{\circ} \mathrm{C}$ is
(a) 42
(b) 62
(c) 74
(d) 104
51. If $q_{w}^{\prime \prime}=5000$ and the convection heat transfer coefficient at the pipe outlet is $1000 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$, the temperature in ${ }^{\circ} \mathrm{C}$ at the inner surface of the pipe at the outlet is
(a) 71
(b) 76
(c) 79
(d) 81

LINKED ANSWER QUESTIONS
Statement for Linked Answ er Questions 52 and 53 :
In orthogonal turning of a bar of 100 mm diameter with a feed of $0.25 \mathrm{~mm} / \mathrm{rev}$, depth of cut of 4 mm and cutting velocity of $90 \mathrm{~m} / \mathrm{min}$, it is observed that the main (tangential) cutting force is perpendicular to the friction force acting at the chip-tool interface. The main (tangential) cutting force is 1500 N .
52. The orthogonal rake angle of the cutting tool in degree is
(a) zero
(b) 3.58
(c) 5
(d) 7.16
53. The normal force acting at the chip-tool interface in N is
(a) 1000
(b) 1500
(c) 2000
(d) 2500

Statement for Linked Answ er Questions 54 and 55 :
In a simple Brayton cycle, the pressure ratio is 8 and temperatures at the entrance of compressor and turbine are 300 K and 1400 K respectively. Both compressor and gas turbine have isentropic efficiencies equal to 0.8 . For the gas, assume a constant value of $c_{p}$ (specific heat at constant pressure) equal to $1 \mathrm{~kJ} / \mathrm{kgK}$ and ratio of specific heats as 1.4. Neglect changes in kinetic and potential energies.
54. The power required by the compressor in $\mathrm{kW} / \mathrm{kg}$ of gas flow rate is
(a) 194.7
(b) 243.4
(c) 304.3
(d) 378.5
55. The thermal efficiency of the cycle in percentage (\%) is
(a) 24.8
(b) 38.6
(c) 44.8
(d) 53.1

## GENERAL APTITUDE (GA) QUESTIONS

Q. 56 to Q. 60 carry one mark each.
56. Complete the sentence

Universalism is to particularism as diffuseness is to
(a) specificity
(b) neutrality
(c) generality
(d) adaptation
57. Were you a bird, you $\qquad$ in the sky
(a) would fly
(b) shall fly
(c) should fly
(d) shall have flown
58. Which one of the following options is the closest in meaning to the word given below?
Nadir
(a) Highest
(b) Lowest
(c) Medium
(d) Integration
59. Choose the grammatically INCORRECT sentence
(a) He is of Asian origin
(b) They belonged to A frica
(c) She is an E uropean
(d) They migrated from India to Australia
60. What will be the maximum sum of $44,42,40$, ?
(a) 502
(b) 504
(c) 506
(d) 500

## Q. 61 to Q. 65 carry two marks each.

61. Out of all the 2-digit integers between 1 and 100, a 2-digit number has to be selected at random. What is the probability that the selected number is not divisible by 7 ?
(a) $13 / 90$
(b) $12 / 90$
(c) $78 / 90$
(d) $77 / 90$
62. A tourist covers half of his journey by train at $60 \mathrm{~km} / \mathrm{h}$, half of the remainder by bus at $30 \mathrm{~km} / \mathrm{h}$ and the rest by cycle at $10 \mathrm{~km} / \mathrm{h}$. The average speed of the tourist in $\mathrm{km} / \mathrm{h}$ during his entire journey is
(a) 36
(b) 30
(c) 24
(d) 18
63. Find the sum of the expression $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}++\frac{1}{\sqrt{80}+\sqrt{81}}$
(a) 7
(b) 8
(c) 9
(d) 10
64. The current erection cost of a structure is `13,200 . If the labour wages per day increase by \(1 / 5\) of the current wages and the working hours decrease by \(1 / 24\) of the current period, then the new cost of erection in` is
(a) 16,500
(b) 15,180
(c) 11,000
(d) 10,120
65. After several defeats in wars, Robert Bruce went in exile and wanted to commit suicide. Just before committing suicide he came across a spider attempting tirelessly to have its net. Time and again, the spider failed but that did not deter it to refrain from making attempts. Such attempts by the spider made Bruce curious. Thus Bruce started observing the nearimpossible goal of the spider to have the net. Ultimately, the spider succeeded in having its net despite several failures. Such act of the spider encouraged Bruce not to commit suicide. And then Bruce went back again and won many a battle, and the rest is history.
Which of the following assertions is best supported by the above information?
(a) Failure is the pillar of success
(b) Honesty is the best policy
(c) Life begins and ends with adventures
(d) No adversity justifies giving up hope

## ANSWERS

| 1. (d) | 2. (c) | 3. (d) | 4. (a) | 5. (b) | 6. (c) | 7. (a) | 8. (c) | 9. (c) | 10. (a) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11. (a) | 12. (c) | 13. (c) | 14. (c) | 15. (d) | 16. (b) | 17. (b) | 18. (b) | 19. (b) | 20. (a) |
| 21. (c) | 22. (c) | 23. (c) | 24. (b) | 25. (c) | 26. (a) | 27. (c) | 28. (a) | 29. (b) | 30. (d) |
| 31. (a) | 32. (b) | 33. (b) | 34. (b) | 35. (b) | 36. (b) | 37. (d) | 38. (c) | 39. (c) | 40. (d) |
| 41. (b) | 42. (a) | 43. (d) | 44. (b) | 45. (d) | 46. (b) | 47. (c) | 48. (c) | 49. (c) | 50. (b) |
| 51. (d) | 52. (a) | 53. (b) | 54. (c) | 55. (a) | 56. (a) | 57. (a) | 58. (b) | 59. (c) | 60. (c) |
| 61. (d) | 62. (c) | 63. (b) | 64. (b) | 65. (a) |  |  |  |  |  |

## EXPLANATIONS

1. Order is index of a derivative present in a PDE (i.e.) maximum there.

Hence, $\frac{\partial^{2} u}{\partial \mathrm{n}^{2}} \Rightarrow$ highest index $2 \Rightarrow$ order $=2$
if $y, y_{2}$ solution of a PDE, then PDE is linearly independent.

$$
\begin{align*}
& \frac{\partial \mathrm{y}_{1}}{\partial \mathrm{t}}+\frac{\mathrm{y}_{1} \partial \mathrm{y}_{1}}{\partial \mathrm{tx}}=\frac{\partial \mathrm{y}_{1}^{2}}{\partial \mathrm{x}^{2}}  \tag{1}\\
& \frac{\partial \mathrm{y}_{2}}{\partial \mathrm{t}}+\frac{\mathrm{y}_{2} \partial \mathrm{y}_{2}}{\partial \mathrm{x}}=\frac{\partial^{2} \mathrm{y}_{2}}{\partial \mathrm{x}^{2}} \tag{2}
\end{align*}
$$

If $y=a y_{1}+a_{1} y_{2}$ is a sum of the PDE
$\frac{\partial}{\partial t}\left(a y_{1}+a_{1} y_{1}\right)+\left(a y_{1}+a_{2} y_{2}\right) \frac{\partial}{d_{2}}\left(a y_{1}+a y_{2}\right)=\frac{\partial^{2}}{\mathrm{~d} 2^{2}}\left(a y_{1}+a y_{2}\right)$
Hence PDE is non linear of order 2
2. Suppose the eigen value of matrix $A$ is $n$

$$
\left(=\lambda+i B_{B}\right) \text { say. }
$$

and the eigen vector is $n$ whereas the conjugate pair of eign value and eign vector is $\lambda$ and $\bar{n}$

$$
\begin{align*}
\text { So, } A n & =\lambda n  \tag{1}\\
\text { And } A \bar{n} & =\lambda \bar{x} \tag{2}
\end{align*}
$$

Taking transpose of equation

$$
\begin{equation*}
x^{-\top} A^{\top}=\bar{x}^{\top} \bar{\lambda} \tag{3}
\end{equation*}
$$

$\Rightarrow \quad x^{-\top} A^{\top} x=x^{-\top} \lambda n$
$\Rightarrow \quad x^{-\top} A n=x^{-\top} \lambda n$
$\Rightarrow \quad \lambda=\bar{\lambda}$
$\Rightarrow \quad \alpha+i \beta=\alpha-i \beta$
$\Rightarrow \quad 2 i \beta=0$
$\Rightarrow \quad \beta=0$
Hence eigen value of a symmetric matrix is real.
3. Trapezoidal rule fits linear polynomial (Ist order) simplex $1 / 3$ rule fits a parabolic curve (2nd order polynomial slip on $3 / 5$ rule fit a able curve (3rd order polynomial
$\therefore$ Correct Answer $\Rightarrow \mathrm{D}$
4. At section - 55

The left side of rod


N ormal stress $\sigma_{n}=\frac{R}{A}=\frac{P}{A}\binom{\varepsilon F n=0}{R-P=0}$
The right side of rod


$$
\begin{array}{rlrl} 
& & \Sigma f x & =0 \\
\Rightarrow & R_{2}-P & =0 \\
\Rightarrow & R_{2} & =P \\
& & & n \\
& & =\frac{P_{2}}{A}=\frac{P}{A}
\end{array}
$$

$\therefore$ Hence normal $=\frac{P}{A}$
5. $d_{B}=2 d_{A}\left[\because U \alpha \frac{1}{A}\right]$
6. $2 \mathrm{vw}=2 \times 0.75 \times 2=3 \mathrm{~m} / \mathrm{s}^{2}$
7. Assumption:
(i) Flow is steady y(i.e.) $\frac{\partial}{\partial t}()=0$
(ii) Fully developed the $\frac{\partial}{\partial t}()-0$; properties are not changing in the direction of the flow.

$\rightarrow$ Pressure is constant along the vertical axis.
$\rightarrow$ Pressure along horizontal axis does change.
$\Delta$ P. $\mathrm{P}_{2} . \mathrm{P}_{1}<0$
Apply N $2 \mathrm{M}\left(2^{\text {nd }}\right)$ over the length I
$\Rightarrow P_{1} \pi r^{2}-\left(P_{1}-|\Delta P|\right) \pi r^{2}-2 \pi r I_{i c}$

$$
\frac{\Delta P}{L}=\frac{2 \tau}{\underset{1}{6 L}}
$$

$N$ either $P$ nor I depend as on $r$.

So, $\frac{\tau}{\varepsilon}$ is independ at then ( $t=r$ where (is on stat)
At center $\mathrm{r}=0, \tau=\mathrm{c} \times 0=0$

$$
\tau=\frac{2 \tau \mathrm{wr}}{\mathrm{D}}
$$

$$
\frac{\Delta \mathrm{P}}{\mathrm{~L}}=\frac{2}{r} \times \frac{\tau \mathrm{W} \times r}{\mathrm{D}} \Rightarrow \mathrm{TW}=\frac{\Delta \mathrm{PD}}{4 \mathrm{~L}}
$$

8. $\mathrm{RH}=0.7=\frac{\mathrm{pv}}{4.25}$ and $\mathrm{w}=\frac{0.622 \mathrm{pv}}{100-\mathrm{pv}}$
9. 

$$
k=k_{0}+b_{T}, k=f(T)
$$

From energy equation:

$$
\begin{align*}
\frac{d}{d x}\left(\frac{k d T}{d x}\right) & =0 \\
\int \frac{d}{a n}\left(\frac{k d T}{d x}\right) & =\int 0 \Rightarrow \frac{k d T}{d x} \cdot C  \tag{1}\\
\int K(T) d T & =\int C \cdot d x \\
\int\left(k_{0}+b_{T}\right) d T & =C x+A
\end{align*}
$$

A is integration constant

$$
\left(K_{0} T+\frac{b T^{2}}{2}\right)=C x+A
$$

Using B.C's

$$
\begin{array}{r}
x=0, T=0 \\
x=1, T=100 \\
A=0 \\
k_{0} \times(100)+\frac{b(10)^{4}}{2}=C \\
\left(100 k_{0}+5000 \mathrm{~b}\right)=C \\
\text { from equation (1) } \frac{d T}{d x}=\left[\frac{100 k_{0}+5000 \mathrm{~b}}{\mathrm{k}_{0}+b_{T}}\right]
\end{array}
$$

10. 



The material (metals) is subjected to high compressive stress as a resort of triction between the roll and the metal suface.
11. P:Friction welding:
(4) J oining of cylindrical dissimilar material.
Q: Gas metal are welding:
(3) Consumable electrode wire
R :Tungsten Inert gas welding: (1) Non-consumable electrode
S : Electro and lag welding: (2) J oining of thick ploter.
12. (c) $\mathrm{d}=\frac{\mathrm{p}}{\tan \theta}$
13.

$$
\begin{aligned}
\lambda & =50, \mu=60 \\
\mathrm{t} & =\frac{\lambda}{\mu(\mu-\lambda)}=.0833 \mathrm{hr} \\
& =5 \mathrm{~min} .
\end{aligned}
$$

14. We know that

$$
\begin{aligned}
f_{t} & =f_{t-1}+\alpha\left(D_{t}-f_{t-1}\right) \\
& =f_{t-1}(1-2)+D_{t}
\end{aligned}
$$

For,

$$
\mathrm{f}_{\mathrm{t}}=\mathrm{D}_{\mathrm{t}}
$$

$$
1-\alpha=0
$$

$$
\Rightarrow \quad \alpha=1
$$

Conclusion ${ }^{\text {a }}$
It is showing the limit of responsiveness
15.
$M R R=\pi D N d f$
16. $\quad t \alpha(V / A)^{2}$ and $m_{2}=8 m_{1}$,

$$
\begin{aligned}
& \alpha a^{2} \text { or } \quad \rho V_{2}=8 \rho V_{1} \Rightarrow V_{2}=8 V_{1} \\
& a_{2}=2 a_{1} \Rightarrow t_{2}=4 t_{1}
\end{aligned}
$$

17. F or a ductive material toughness is a measure of ability to absorb energy up to fracture

(a) Resistance to scrateling is resistance to abrasion.
(b) Toughness is the ability to absorb energy upto fracture.
(c) Proof resilence is the ability to absorb energy till elastic limit.
(d) Hardness the resistance to indentation.
18. Weknow $\Rightarrow$ [Power $=$ Force $\times$ Velocity $]$

$$
\begin{align*}
\mathrm{f} & =-\mathrm{m}\left[\mathrm{v}_{\mathrm{f}}-\mathrm{v}_{\mathrm{i}}\right] \\
\mathrm{f} & =-\mathrm{fQ}\left[\left(-\mathrm{v}_{\mathrm{i}}+2 \mathrm{u}\right)-\mathrm{v}_{\mathrm{i}}\right] \\
\mathrm{f} & =-\mathrm{fQ}\left(-2 \mathrm{v}_{\mathrm{i}}+2 \mathrm{u}\right) \\
& \therefore f=2 f \mathrm{Q}\left[-v_{i}-u\right] \tag{1}
\end{align*}
$$

So,

$$
P=2 P Q\left(v_{i}-u\right) \cdot \mu
$$

To find speed at maximum power:

$$
\begin{aligned}
\frac{d p}{d u}=0, \frac{d p}{d u} & =2 f Q\left[\left(v_{i}-u\right)+u(-1)\right] \\
& =2 f Q\left[v_{i}-2 u\right] \\
\frac{d p}{d u} & =0, \\
\Rightarrow \quad v_{i}=2 n \quad & \frac{u=v_{i}}{2}
\end{aligned}
$$

19. 

$$
\begin{aligned}
\frac{d^{2} t}{d x^{2}}+\frac{q}{x} & =0 \\
\frac{\mathrm{dT}}{\mathrm{dx}} & =\frac{-\mathrm{q}}{\mathrm{k}} \mathrm{x}+\mathrm{c}_{1}
\end{aligned}
$$

$$
1=\frac{-q}{k} \frac{x^{2}}{2}+c_{1} x+c_{2} \quad \text { (parabolic) }
$$

at

$$
x=0, T_{1}=0
$$

$$
x=I, T_{2}=100
$$

$$
0=0+c_{2} \Rightarrow c_{2}=0
$$

$$
100=\frac{-\mathrm{qt}^{2}}{2 \mathrm{k}}+\mathrm{c}_{1} \mathrm{l}
$$

$$
\frac{100}{\mathrm{~L}}-\frac{\mathrm{qL}}{2 \mathrm{~K}}=\mathrm{C}_{1}
$$

$$
\mathrm{T}=\frac{-q}{k} \frac{x^{2}}{2}+\left(\frac{100}{l}-\frac{q l}{2 k}\right) x
$$

For maximum temp:

$$
\frac{d T}{d x}-0 \Rightarrow \frac{-q \times L x}{2 x}+\frac{100}{L} \frac{q L}{2 x}=0
$$

20. $\mathrm{P}_{1} \mathrm{~V}_{1} \ln p_{2 / p_{1}}=W_{\text {isothermal }}$
21. 
22. 



The whiring frequency of shaft is given by

$$
\begin{aligned}
\mathrm{f} & =\frac{\pi}{2} n^{2} \sqrt{\frac{g \mathrm{EI}}{w L^{4}}} \\
\mathrm{f}_{1} & =\frac{\pi}{2} \sqrt{\frac{g \mathrm{EI}}{w L^{4}}}
\end{aligned}
$$

$\mathrm{n} \Rightarrow$ node number
frequency of 1st mode
2 nodes are present in 3 rd mode.

$$
f_{3}=3^{2} f_{1}=1800(r p m)
$$

$$
\Rightarrow \quad \mathrm{f}_{1}=\frac{1800}{9}=200(\mathrm{rpm})
$$

$$
\begin{aligned}
& \underline{6 h}=\frac{P D}{2 t} \text { where } P=\text { internal pressure } \\
& 61=\frac{P D}{4 t} \quad D=\text { inner dia of cylinder. } \\
& t=\text { thickness of cylinder. } \\
& \frac{6 h}{6 l}=\frac{\frac{P D}{2 t}}{\frac{P D}{4 t}}=2
\end{aligned}
$$

23. The link opposite to shortest link is fixed.
24. $\mathrm{P}(\mathrm{x}<0)=\mathrm{P}\left\{\frac{(\mathrm{X}-\overline{\mathrm{X}})}{\sigma}<\frac{0-\overline{\mathrm{X}}}{\sigma}\right\}$


$$
\begin{array}{ll}
\because & \overline{\mathrm{X}}=1, \sqrt{V}=\sigma \text { or } \sigma=\sqrt{4}=2 \\
\therefore & \mathrm{P}\left\{\frac{\mathrm{x}-1}{2}<-\frac{1}{2}\right\} \text { or } \mathrm{P}\left\{\mathrm{z}<-\frac{1}{2}\right\} \\
& \mathrm{P}\left(\mathrm{z}<-\frac{1}{2}\right)=\mathrm{P}(\mathrm{z}>1 / 2)
\end{array}
$$

$\because P(x>a)$ represents area under curve for $x>a$ 25. Going by options,

$$
\begin{aligned}
\cos 2 x=2 \cos ^{2} x-1 & =2 \cos ^{2} x-\left(\sin ^{2} x+\cos ^{2} x\right) \\
\cos 2 x & =\cos ^{2} x-\sin ^{2} x
\end{aligned}
$$

26. U se Gauss-Divergence theorem,

$$
\begin{aligned}
\mathrm{I}=\iint_{s} \frac{1}{4} \overrightarrow{\mathrm{~F}} \cdot \hat{\mathrm{n} d A} & =\iiint_{V} \vec{\nabla} \cdot \overrightarrow{\mathrm{~F}} \mathrm{dV} \\
\vec{\nabla} \cdot \overrightarrow{\mathrm{~F}} & =1+1+1=3 \\
\therefore \quad & \\
I & =\frac{1}{4} \times 3 \mathrm{~V}=\frac{3}{4} \times \frac{4}{3} \pi(1)^{3} \\
& =\pi
\end{aligned}
$$

27. Check by options

$$
L^{-1}\left\{\frac{4}{s^{2}+1}\right\}=4 \text { sint satisfies } \frac{d^{2} f}{{d t^{2}}^{2}}+f=0
$$

33. 
34. 

$$
\begin{array}{lr}
\Rightarrow & \mathrm{T}=15 \\
\therefore & \mathrm{~V}(15)^{1.6}=3000 \text { for carbide }
\end{array}
$$

35. 


$L^{-1}\left\{\frac{4}{s^{2}+1}\right\}=4 \sin$ satisfies $\frac{d^{2} f}{d t^{2}}+f=0$

$$
E F=P F \cos 450=\frac{5}{\sqrt{2}}
$$

N ow for mirror image: $P F=F P^{1}=\frac{5}{\sqrt{2}}$ and $E F=\frac{\mathrm{J}}{\sqrt{2}}$
28. $\dot{\mathrm{Q}}-\dot{\mathrm{W}}=\dot{\mathrm{m}}\left[\left(\mathrm{h}_{2}-\mathrm{h}_{1}\right)+\left(\frac{\mathrm{V}_{2}^{2}-\mathrm{V}_{1}^{2}}{2}\right)\right]$
29.
and $\dot{\theta}=-5 \mathrm{KW} \quad \mathrm{m}=1 \mathrm{~kg} / \mathrm{s}$
$=A_{1} V_{1}=A_{2} V_{2}$

$$
\begin{aligned}
\Rightarrow \quad \frac{\pi}{4}(20)^{2} \times 2 & =\frac{\pi}{4} \times d^{2} \times \sqrt{2 \times 9.8 \times 0.5} \\
d & \approx 15 \mathrm{~mm}
\end{aligned}
$$

30. $\quad \frac{\mathrm{T}-\mathrm{T}_{\infty}}{\mathrm{T}_{\mathrm{i}}-\mathrm{T}_{\infty}}=e^{-\frac{h \mathrm{~A}}{\rho \mathrm{~V}_{c}}}$ or $0.4=e$

$$
\frac{h \mathrm{~A}}{\rho \mathrm{~V} c}=\frac{h(b)}{\rho(\mathrm{D}) c} \because \frac{\mathrm{~V}}{\mathrm{~A}}=\frac{4 / 3 \pi r^{3}}{4 \pi r^{2}}=\frac{r}{3}=\frac{D}{6}
$$

31. 

$$
\Delta \mathrm{E}=\mathrm{I} \mathrm{w}_{\mathrm{m}}^{2} \mathrm{c}_{\mathrm{f}}
$$

32. 

$$
\begin{aligned}
m & =\frac{D}{T} \text { and } m_{p}=m_{Q} \\
m_{R} & =m_{s} \text { for gear meshing condition }
\end{aligned}
$$

$$
\begin{gathered}
\therefore F P^{1}=\sqrt{E F^{2}+F P^{12}}=\sqrt{\left(\frac{5}{\sqrt{2}}\right)^{2}+\left(\frac{5}{\sqrt{2}}\right)^{2}} \\
E P^{1}=5
\end{gathered}
$$

N ow from symmetrically we can calculate the y position (i.e.) $P^{\prime} D=5$
36.
$\therefore \quad p^{1}$ will be at $(10,5)=(10,5)$

37. Go gauge measures maximum material limit

$$
\begin{array}{rlrl}
\therefore \quad 25.010 & +0.032=25.042 \\
& m & \times q \\
m & =Z q \\
m & =Z i t \\
\therefore \quad M R R=\frac{m}{p t} & =\frac{Z i}{\rho}=\frac{E i}{\rho F}\left(Z=\frac{E}{F}\right)
\end{array}
$$

38. 

$$
\begin{array}{ll}
\therefore & M R R \times E i \quad E=\frac{\text { At.wt. }}{\text { valency }} \\
\Rightarrow & \frac{(M R R)_{1}}{(M R R)_{2}}=\frac{E_{1} i_{1}}{E_{2} i_{2}} \\
\Rightarrow & \frac{0.26}{(M R R)_{2}}=\frac{28 \times 1000 \times 0.9}{16 \times 2000 \times 0.9} \\
\therefore & (M R R)_{2}=0.297 \approx 0.30 \mathrm{gm} / \mathrm{s}
\end{array}
$$

39. 

$\frac{1}{2} \mathrm{mv}^{2}=\frac{1}{2} \mathrm{kx}^{2}$ and $\int \mathrm{F} d t=m\left(v_{2}-v_{1}\right)$

$$
\begin{array}{ll}
\Rightarrow & x=\frac{v}{\sqrt{\mathrm{~K}}}=\frac{0.5}{\sqrt{10000}} \Rightarrow 5 \times 10^{3} \times 10^{-4}=1 \times v \\
\Rightarrow & v=0.5 \mathrm{~m} / \mathrm{s}
\end{array}
$$

40. Soderberg criteria is:

$$
\begin{aligned}
\frac{\sigma_{\mathrm{a}}}{\sigma_{\mathrm{e}}}+\frac{\sigma_{\mathrm{m}}}{\mathrm{~S}_{\mathrm{y}+}} & =\frac{1}{\mathrm{n}} \text { and } \sigma_{\mathrm{a}}=\frac{100-20}{2 \mathrm{~A}} \\
\Rightarrow \quad \frac{40}{160}+\frac{60}{240} & =\frac{1}{2} \times \mathrm{A} \times 10^{3} \quad \sigma_{m}=\frac{100+20}{2 \mathrm{~A}}
\end{aligned}
$$

$$
\mathrm{A}=1000 \mathrm{~mm}^{2}
$$

41. 



Total load $=\int_{0}^{2} \sin \left(\frac{3 n x}{\mathrm{~L}}\right) \mathrm{dx} P=-\frac{\mathrm{L}}{3 \pi}\left[\cos \left(\frac{3 \pi x}{\mathrm{~L}}\right)\right]_{0}^{\mathrm{L}}=\frac{2 \mathrm{~L}}{3 \pi}$
By symmetry, $R_{1}=R_{2}=P / 2$
42. $\quad \mathrm{q}_{\mathrm{T}-2}=\frac{0\left(\mathrm{~T}_{1}^{4}-\mathrm{T}_{2}^{4}\right)}{1 / \varepsilon_{1}+1 / \varepsilon_{2}-1}$

$$
\begin{aligned}
& =\frac{5.67 \times 10^{-8}\left(400^{4}-300^{4}\right)}{\left(2 / 8^{-1}\right)} \\
& =0.66 \frac{\mathrm{kw}}{\mathrm{~m}^{2}}
\end{aligned}
$$

43. 

$$
\int_{0}^{L} e g \sin \theta s^{2} d=m g \frac{1}{2} \cos \theta
$$

$$
\begin{aligned}
m & =\frac{2 e^{2} \tan \theta}{3} \\
& =\frac{2}{3} \times 10^{3} \times 5^{2} \times \tan 30^{\circ}=9653 x y .
\end{aligned}
$$

53. $\mathrm{F}_{\mathrm{c}}=\mathrm{N}$


$$
\begin{aligned}
W_{c} & =\left(h_{2}-h_{1}\right) \\
\overline{\mathrm{m}} & =C_{p}\left(T_{2}-T_{1}\right)=C_{p} \frac{\left(T_{2}^{1}-T_{1}\right)}{m_{c}} \\
\&\left(T_{2}^{1} / T_{1}\right) & =(r p) \frac{V-1}{V}
\end{aligned}
$$

55. 

$$
\eta=\frac{W_{T}-W_{c}}{Q_{\text {in }}}=\frac{\left(T_{3}-T_{4}\right)-\left(T_{2}-T_{1}\right)}{\left(T_{3}-T_{2}\right)}
$$

\& for turbine, $=\eta_{T}=\frac{T_{3}-T_{4}}{T_{3}-T_{4^{1}}}$

$$
\& \frac{T_{3}}{T_{4^{1}}}=\left(r_{p}\right)^{(r-1)} r
$$

56. Universalism is to particularism $\leftarrow$ global consideration to a perticular one is moving towards oposite

## $\Downarrow$

difference is to- <opposite the idea of differnece which pertains to reaching out to surrounding environment opposite idea to differences is specify.
60.

$$
\begin{aligned}
\mathrm{S}_{\mathrm{n}}= & \frac{\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}] \\
& 44,42,40,-----2,0 \\
\mathrm{n} & =22 \text { terms } \\
\mathrm{P}(\mathrm{~A})= & 1-\mathrm{P}(\overline{\mathrm{~A}}) \\
= & 1-\frac{13}{90}=\frac{77}{90} \\
\mathrm{~V}_{\mathrm{avg}}= & \frac{\text { Total distance }}{\text { Total time }}
\end{aligned}
$$

61. 
62. 
63. Multiply each term and divide each term by its conjugate.
64. 

Cost $\alpha$ (wages $\times$ hours) c $\alpha$ wh

