

(3 Hours)

[Total Marks : 100]

- B. 1. Question No. 1 is compulsory
 2. Attempt any FOUR from remaining six questions.
 3. Figures to the right indicate the full marks.
 4. Assume the suitable data if needed with justification.

- Q.1 a. Show that $L\left\{\frac{\cos \sqrt{t}}{\sqrt{t}}\right\} = \sqrt{\frac{\pi}{s}} e^{\frac{-1}{4s}}$ 05
- b. For $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 2 & 2 & 0 \end{bmatrix}$ verify $A(\text{adj } A) = |A| I$ 05
- Hence find inverse of A
- c. Show that $f(z) = \bar{z}$ is not analytic. 05
- d. $\int_0^{1+i} (x^2 - iy) dz$ along the path (i) $y = x$ (ii) $y = x^2$ 05
- Q.2 a. Prove that $u(x, y) = x^2 - y^2$ and $v(x, y) = \frac{y}{x^2 + y^2}$ are both harmonic functions but $u + iv$ is not analytic. 06
- b. Prove that every square matrix can be uniquely expressed as the sum of the hermitian matrix and a skew-hermitian matrix. 06
- c. Find 08
- (i) $L\{\cosh 2t \cdot \text{erf} 3\sqrt{t}\}$ if $L\{\text{erf}(\sqrt{t})\} = \frac{1}{\sqrt{s+1}+1}$
- (ii) $L\left\{\frac{e^{2t} \sin t}{t}\right\}$
- Q.3 a. Show that $\int_C \frac{dz}{z+1} = 2\pi i$ where C is the circle $|z| = 2$ 06
- Hence deduce that $\int_C \frac{(x+1)dx + ydy}{(x+1)^2 + y^2} = 0$ and $\int_C \frac{(x+1)dy - ydx}{(x+1)^2 + y^2} = 2\pi$
- b. Find the eigen values and eigen vectors of 06
- $$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$
- c. Find the inverse Laplace of 08
- (i) $\frac{3s+1}{(s+1)(s^2+2)}$ (ii) $\log\left(\frac{s^2+a^2}{s^2+b^2}\right)$

Q.4 a. Find the Laplace transformation of the function

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$$f(t) = \begin{cases} (t-a)^3 & t > a \\ 0 & t < a \end{cases}$$

b. Find the constant 'a' so that the function $u = x^2 + ay^2$ is harmonic.

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Also find the harmonic conjugate.

c. Find the values of λ and μ such that the following equations

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$$2x + 3y + 5z = 9$$

$$7x + 3y - 2z = 8$$

$$2x + 3y + \lambda z = \mu$$

has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

Q.5 a. Find the Laplace transformation of the following function,

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$$\begin{aligned} f(t) &= \frac{t}{a} & 0 < t < a \\ &= \frac{1}{a}(2a-t) & a < t < 2a \end{aligned}$$

Where $f(t) = f(t+2a)$ b. If $A = \begin{bmatrix} \sin \theta & \cos \theta & 1 \\ \sec \theta & \cos \theta & 1 \\ \tan \theta & \cot \theta & 1 \end{bmatrix}$ then prove that

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there does not exist a real value of θ for which the characteristic roots are -1, 1, 3.c. Obtain two distinct Laurent's series of $f(z) = \frac{1}{z^2(2-z)}$ about $z = 0$.

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Q.6 a. Define orthogonal matrix and unitary matrix

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Prove: If 'A' and 'B' are two orthogonal matrices of order n then AB and BA are also an orthogonal matrices.

b. If $f(\xi) = \int_{-\xi}^{\xi} dz$ where C is $x^2 + y^2 = 4$

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Find the values of (i) $f(3)$, (ii) $f'(1-i)$, (iii) $f''(1-i)$, ...c. Solve $\frac{dy}{dx} + 2\frac{dy}{dx} - 3y = 0$ given, at $x=0, y=0$ and $\frac{dy}{dx} = 4$

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Q.7 a. Verify Cayley-Hamilton thm. For $A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ hence obtain

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 A^{-1} if exists

b. State and prove Cauchy's Thm.

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Find the bilinear transformation which maps the points $-1, 1, \infty$ to the points $i, -i, j$. Also find its fixed points.

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