RW-6131 **511101**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

ALGEBRA—I

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** questions.

1. If $a \in G$, define $N(a) = \{x \in G \mid xa = ax\}$, show that $N(a)$ is a subgroup of G.

2. Give an example of an isomorphism of a group G into itself which is not onto.

3. Find $p(5)$.

4. Prove that the conjugacy is an equivalence relation on G.

5. Prove that any field is an integral domain.

6. If U is an ideal of R and $1 \in U$, prove that $U = R$.

7. Find all the units in $J[i]$.

8. Prove that the polynomial $f(x) = x^4 + 2x + 2$ is irreducible over the field of rational numbers.

9. Prove that direct sum of free modules is a free module.

10. Define a maximal submodule and a simple module.

Part - B (5 × 5 = 25)

Answer **all** questions.

11. (a) Let ϕ be a homomorphism of G onto $\bar{G}$ with kernel K. Prove that $G/K \approx \bar{G}$.

Or

(b) Prove that $I(G) \approx G/Z$ where I (G) is the group of inner automorphisms of G and Z is the centre of G.

12. (a) If $0(G) = p^2$ where p is a prime number, prove that $Z(G) \neq (e)$.

Or

(b) Prove that any group of older 72 cannot be simple.

13. (a) Prove that a finite integral domain in a field.

Or

(b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Prove that R is a field.

14. (a) Let R be a Euclidean ring. Prove that any two elements a and b in R have a g.c.d. d with $d = \lambda a + \mu b$ for some $\lambda, \mu \in R$.

Or

(b) State and prove Gauss' lemma.

15. (a) Give an example of a free module M for which a subset X spans M but X does not contain a basis.

Or

(b) State and prove Schur's lemma.

Part - C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) Prove that HK is a subgroup of G if and only if HK = KH.

(b) If H and K are finite subgroups of G of order $0(H)$ and $0(K)$ respectively, prove that $0(HK) \cdot 0(H \cap K) = 0(H) \cdot 0(K)$.

17. State and prove the second and third parts of Sylow's theorem.

18. Prove that every integral domain can be imbedded in a field.

19. (a) If $f(x)$ and g (x) are primitive polynomials prove that $f(x)$ g (x) is a primitive polynomial.

(b) State and prove Eisenstein's criterion

20. (a) Prove that a vector space is a free module.

(b) Prove that an R-module M is simple if and only if M $\approx$ R/I for some maximal left ideal I in R.

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RW-6132 **511102**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

REAL ANALYSIS

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** the questions.

1. State the least upper bound property and the greatest lower bound property in an ordered set.

2. Prove that $|x+y| \le |x| + |y|$ for any $x, y \in R^k$

3. Define an uniformly continuous function.

4. Give an example of a function f which has a discontinuity of the second kind at every point x.

5. Give an example of a continuous function which is not differentiable.

6. Define a local minimum of a function f at a point p.

7. Define the Riemann-Stieltjes integral.

8. Define a curve and rectifiable curve.

9. Define an uniformly convergence sequence of functions.

10. Give a sequence of functions which is not equi-continuous.

Part - B (5 × 5 = 25)

Answer **all** questions.

11\. (a) Suppose S is an ordered set with the least-upper-bound property. BCS is not empty, and B is bounded below. Let L be the set of all lower bounds of B. Then prove that α = sup L exists in S and α = inf B exists in S.

Or

(b) State and prove the Schwarz inequality.

12\. (a) Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Prove that f (X) is compact.

Or

(b) let f be monotonic on (a, b). Show that the set of points of (a, b) at which f is discontinuous is at most countable.

13. (a) Let f be defined on $[a, b]$. If f is differentiable at a point $x \in [a, b]$, then show that f is continuous at x.

Or

(b) State and prove the generalized mean value theorem.

14. (a) If P^* is a refinement of P, prove that

$$L(P, f\alpha) \leq L(P^*, f, \alpha) \text{ and } U(P^*, f, \alpha) \leq U(P, f, \alpha)$$

Or

(b) State and prove the fundamental theorem of Calculus.

15. (a) State and prove the Cauchy criterion for uniform convergence.

Or

(b) Prove that there is a real continuous function on the real line which is nowhere differentiable.

Part - C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) State and prove the Archimedean property of R.

(b) Show that between any two real numbers there is a rational number.

17. Let f be a continuous mapping of a contact metric space X into a metric space Y. Prove that f is uniformly continuous on X.

18. State and prove the Chain rule for differentiation.

19. Show that $f \in \mathscr{R}(\alpha)$ on $[a, b]$ if and only if for every $\varepsilon > 0$, there is a partition P such that

$$U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$$

20. State and prove the Stone-Weierstrass theorem.

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RW-6133 **511103**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

DIFFERENTIAL EQUATIONS

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** questions.

1. Verify $\phi_1(x) = x^2$ $(x > 0)$ is a solution of $x^2 y'' - 7x y' + 15 y = 0$. Find a second independent solution.

2. Write the values of the Legendre polynomials $P_3(x)$ and $P_4(x)$.

3. Find the singular points and its nature for the equation $3x^2 y'' + x^6 y' + 2x y = 0$.

4. Compute the indicial polynomial and its roots of the equation $x^2 y'' + xy' + (x^2 - \alpha^2) y = 0$ where α is a non-negative constant.

5. Eliminate the arbitrary function f from the equation $z = x y + f(x^2 + y^2)$.

6. Find a complete integral of the equation $(p^2 + q^2)(z - xp - yq) = 1$.

7. If $u = f(x + iy) + g(x - iy)$ where the functions f and g are arbitrary, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

8. Find a particular integral of the equation $(D^2 - D')z = 2y - x^2$.

9. Prove that $r \cos\theta$ and $r^{-2} \cos\theta$ satisfy the Laplace equation, where r, θ, ϕ are spherical polar co-ordinates.

10. Define exterior Churchil problem.

Part - B (5 × 5 = 25)

Answer **all** questions.

11. (a) If $\phi_1, \phi_2, ..., \phi_n$ are n solutions of $L(y) = 0$ on an interval I, prove that they are linearly independent if, and only if, $W(\phi_1, \phi_2, ..., \phi_n)(x) \neq 0$ for all x in I.

Or

(b) Show that $\int_{-1}^{+1} P_n(x) P_m(x)\, dx = \begin{cases} 0 & \text{if } n \neq m \\ \dfrac{2}{2n+1} & \text{if } n = m. \end{cases}$

12. (a) Show that the equation $x y'' + (1 - x) y' + \alpha y = 0$, where α is a constant, has a regular singular point at $x = 0$. Compute the indicial polynomial and its roots. Find a solution ϕ of the form $\phi(x) = x^2 \sum_{k=0}^{\infty} C_k x^k$.

Or

(b) Show that $(x^{\alpha} J_{\alpha})'(x) = x_{\alpha} J_{\alpha\text{-}1}(x)$ and

$(x^{-\alpha} J_{\alpha})'(x) = -x^{-\alpha} J_{\alpha+1}(x)$.

13. (a) Show that the equations $xp = yq, z\,(xp + yq) = 2xy$ are compatible and solve them.

Or

(b) Find the integral surface of the linear partial differential equation $x\,(y^2 + z)\,p - y\,(x^2 + z)\,q = (x^2 - y^2)\,z$ which contains the straight line $x + y = 0, z = 1$.

14. (a) Reduce the equation

$$y^2 \frac{\partial^2 z}{\partial x^2} - 2\,xy \frac{\partial^2 z}{\partial x \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x} \frac{\partial z}{\partial x} + \frac{x^2}{y} \frac{\partial z}{\partial y}$$ to

canonical form and hence solve it.

Or

(b) If the two dimensional harmonic equation $\nabla_1^2 V = 0$ is transformed to plane polar coordinates r and θ, defined by $x = r\cos\theta$, $y = r\sin\theta$ show that it takes the form $\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r2}\frac{\partial^2 V}{\partial \theta^2} = 0$.

Deduce that it has solutions of the form $(Ar^n + Br^{-n})e^{\pm in\theta}$, where A, B and n are constants.

15. (a) Find the potential function $\psi(x, y, z)$ in the region $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$ satisfying the conditions.

$\psi = 0$ on $x = 0, x = a, y = 0, y = b, z = 0$

$\psi = f(x, y)$ on $z = c$, $0 \le x \le a$, $0 \le y \le b$.

Or

(b) Show that y = A (p) $e^{ip\,(t \pm x/c)}$ is a solution of the wave equation for arbitrary forms of the function A which depends only on p. Interpret these solutions physically.

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. (a) State and derive the Rodrigue's formula for the Legendre polynomials $P_n(x)$.

(b) Find two linearly independent solutions in power series of the equation $y'' - x^2 y = 0$.

17. Show that infinity is a regular singular point of the equation $(1 - x^2)\, y'' - 2xy' + \alpha\,(\alpha + 1)\, y = 0$.

Compute the induced equation associated with the above equation and the substitution $x = \frac{1}{t}$. Compute the indicial polynomial, and its roots of the induced equation. Taking $\alpha = 1$, find two linearly independent power series solutions of $(1 - x^2)\, y'' - 2xy' + 2y = 0$.

18. Explain the Charpit's method of solving a given partial differential equation $f(x, y, z, p, q) = 0$. Hence or otherwise solve $p^2 x + q^2 y = z$.

19. By separating the variables, show that

(a) the one dimensional wave equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 z}{\partial t^2}$ has solutions of the form A exp $(\pm in\, x \pm in\, (t))$, where A and n are constants.

(b) the one dimensional diffusion equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k} \frac{\partial z}{\partial t}$ has solutions of the form

$$z(x, t) = \sum_{n=0}^{\infty} c_n \cos(n\,x + t_n)\, e^{-n^2 kt} .$$

20. If a string is released from rest in the position $y = \frac{4\varepsilon}{l^2} x (l - x)$, show that its motion is described by the equation

$$y = \frac{32\,\varepsilon}{3} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^3}$$

$$\sin \frac{(2n+1)\,\pi\, x}{l} \cdot \cos \frac{(2n+1)\,\pi\, c\, t}{l}.$$

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RW-6134 **511501**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

Elective I—ANALYTIC NUMBER THEORY

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** questions.
Each question carries 2 marks.

1. If $(a, b) = 1$, then prove that $(a^n, b^k) = 1$ for all $n \geq 1$, $k \geq 1$.

2. Fin all integers n such that $\phi(n) = n/2$

3. If $x \geq 1$ prove that $\sum_{n > x} \frac{1}{n^3} = 0(x^{1-s})$ if $s > 1$.

4. State the Legendre's Identity

5. Define Chebyshev's - *9* function and state a property of it.

6. With the usual notation define M (x) and H (x) and write down the relation between them.

7. If $a \equiv b$ (mod. m) and if d/m and d/a then prove that d/b.

8. Solve the congruence $5x \equiv 3$ (mod 24).

9. State Gauss lemma.

10. Define the Jacobi symbol.

Part - B (5 × 5 = 25)

Answer **all** questions.

Each question carries 5 marks.

11. (a) State and prove the division algorithm.

Or

(b) Prove that $\sum_{d^2 / n} \mu(d) = \mu^2(n)$

12. (a) State and prove the Euler's summation formula.

Or

(b) For all $x \geq 1$ prove that

$$\left| \sum_{n \leq x} \frac{\mu(n)}{n} \right| \leq 1 \text{ with equality holds only if } x < 2.$$

13. (a) State and prove the Abel's identity.

Or

(b) Let $f(x) = x^2 + x + 41$. Find the smallest integer $x \geq 0$ for which $f(x)$ is composite.

14. (a) State and prove the Chinese reminder theorem.

Or

(b) Find all x which simultaneously satisfy the system of convergences $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 4$, $x \equiv 3 \pmod 5$.

15. (a) State and prove the Euler's Criterion.

Or

(b) If P is an odd positive integer then prove that

$(-1 \mid p) = (-1)^{(p-1)/2}$

Part - C (3 × 10 = 30)

Answer any **three** questions.
Each question carries 10 marks.

16. For $n \geq 1$, prove that $\phi(n) = n \prod_{p/n} \left(1 - \frac{1}{p}\right)$ and state and five properties of ϕ and prove the same.

17. If $x \geq 1$ prove that

(a) $$\sum_{n \leq x} \frac{1}{n} = \log x + C + 0\left(\frac{1}{x}\right)$$

(b) $$\sum_{n \leq x} \frac{1}{n^s} = \frac{x^{1-s}}{1-s} + \zeta(s) + 0(x^{-s})$$ if $s > 0$, $s \neq 1$.

18. Let p_n denote the n^{th} prime. Then prove that the following asymptotic relations are logically equivalent.

(a) $$\lim_{x \to \infty} \frac{\pi(x) \log x}{x} = 1$$

(b) $$\lim_{x \to \infty} \frac{\pi(x) \log \pi(x)}{x} = 1$$

(c) $$\lim_{n \to \infty} \frac{p_n}{n \log n} = 1$$

19. (a) State and prove the Lagrange's Theorem.

(b) State and prove the Wolstenholme's theorem.

20. State and prove the Quadratic reciprocity law.

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RW-6135 **511701**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

I/D—OBJECT ORIENTED PROGRAMMING AND C++

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is single line comment ? Write an example.

2. int * p ; int a, $b = 5$; $p = \& b$; $a = * p$; What is the value of a ?

3. Name two flags used in ios class for formatting numbers.

4. Write a function prototype for a function that receives two numbers adds them and return the result.

5. Write a function prototype to compute simple interest. Provide default arguments for p, n and r.

6. What is the job of destructor ? How it differs from constructor in name ?

7. Write two operators which can't be overloaded.

8. How many operands are passed to an overloaded operator friend function ?

9. What are the two types of polymorphisms ?

10. Write a declaration for two files and open them one for reading and other for writing.

Part - B (5 × 5 = 25)

Answer **all** questions.

11. (a) List the application and benefits of OOP.

(*Or*)

(b) Describe the general structure of a C++ program.

12. (a) Explain different ways of passing arguments to Q function through' examples.

(*Or*)

(b) With an example program, explain the use of friend function. How it differs from member function ?

13. (a) Write a C++ program with parameterised constructor to initialize three variables a, b, c for Quadratic equation.

(*Or*)

(b) What is copy constructor ? Explain its necessity.

14. (a) What is operator overloading ? Write the rules governing operator overloading.

(*Or*)

(b) Write a C++ program that overloads assignment operator.

15. (a) What is inheritance ? Write a program using single in heritance.

Or

(b) Explain any five functions used for file processing.

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. Explain the features and data types of C++.

17. Explain I/O manipulators for I/O formatting.

18. Explain constructor overloading through an example program.

19. Write a C++ program to add two $n \times n$ matrices by overloading ' + ' operator.

20. Write a program that stores objects on file.

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RW-6136 | **511301**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

MECHANICS

(CBCS—2008 onwards)

Time : 3 Hours | Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** questions.
Each question carries 2 marks.

1. Define Holonomic constraints.

2. State the principle of work and kinetic energy.

3. Define a conservative system.

4. State Liouville's system.

5. State the sufficient conditions for a function $f\ (q_1,\ q_2\ .....q_n)$ to have a local minimum at a reference point $\bar{q}_0$.

6. What is Brachistochrone problem ?

7. State the limitations of Stackel's theorem.

8. Define integral's of the motion.

9. What is Poisson Brackets?

10. Define contact transformation.

Part - B (5 × 5 = 25)

Answer **all** questions.
Each question carries 5 marks.

11. (a) State and prove Konig's theorem.

Or

(b) Explain principle of Virtual work.

12. (a) A particle of unit mass moves under attraction to a fixed point 'O' by an inverse square gravitational force. Obtain the integral of motion.

Or

(b) Derive the Lagrange's equation using Routhian function.

13. (a) Find the stationary values of the function $f = z$, subject to the constraints,

$$\phi_1 = x^2 + y^2 + z^2 - 4 = 0,$$

$$\phi_2 = xy - 1 = 0$$

Or

(b) Solve Brachistochrone problem

14. (a) State and prove Jacobi's problem.

Or

(b) Obtain modified Hamilton-Jacobi equation.

15. (a) Explain Lagrange brackets.

Or

(b) Show that the transformation $Q = \frac{1}{2}(q^2 + p^2)$,

$P = -\tan^{-1}\frac{q}{p}$ is canonical.

Part - C (3 × 10 = 30)

Answer any **three** questions.

Each question carries 10 marks.

16. Explain the following :

(a) Angular momentum.

(b) Generalized momentum.

17. A double pendulum consists of two particles suspended by massless rods. Assuming that all motion takes place in a vertical plane, find the differential equations of motion. Linearize these equations, assuming small motions.

18. State and prove Hamilton principle.

19. State and prove Stackel's theorem.

20. Consider the transformation

$Q = q - tp + \frac{gt^2}{2}$; $p = p - gt$ find K-H and generating functions.

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RW-6137 **511302**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

FUNCTIONAL ANALYSIS

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** questions.

1. Show that every normed space is metric space.

2. Is it true that a non-constant linear functional on a normed linear space is onto ? Why ?

3. If $x \perp M$ where M is a total set in an inner product space, what best can you say about x ? Why ?

4. Define a separable topological space.

5. If $\langle u, w \rangle = \langle v, w \rangle$ for all w in an inner product space H ($u, v, \in$ H) then show that $u = v$.

6. Define a sesquilinear form.

7. Give an example of an inner product space which is not a highest space.

8. Describe parallelogram law for normed linear spaces.

9. Define weak convergence of a sequence in a normed linear space.

10. If X and Y are Banach spaces is it true that B (X, Y) is a normed linear space ? If so what is the norm of $\tau \in$ B (X, Y)

Part - B (5 × 5 = 25)

Answer **all** questions.

11. (a) Show that a finite dimensional vector space is algebraically reflexive.

Or

(b) For an operator T on a finite dimensional Hilbert space with values in another Hilbert space show that the range R (T) is also finite dimensional vector space.

12. (a) State and prove Bessel's inequality.

Or

(b) Let $x_n \to x$ in a Hilbert space H and $y \perp x_n$ for each n. Show that $x \perp y$.

13. (a) Show that the limit of a sequence of bounded self adjoint operation is again self adjoint.

Or

(b) Show that every bounded linear operator T on a complex Hilbert space can be decomposed as $T = T_1 + iT_2$ where T_1 and T_2 are uniquely defined self adjoint operators

14. (a) Let X be a normed linear space and $x_0 \neq 0$ be in H. Show that there exists a bounded linear functional f on X with $\| f \| = 1$, $f\ (x_0) = \| x_0 \|$. Deduce that if $f(x) = f(y)$ for every bounded linear functional f on a normed space then $x = y$.

Or

(b) Explain in detail an isomorphism of a normed linear space onto a subspace of its second dual.

15. (a) In a normed linear space show that strong convergence implies weak convergence but not vice-versa. However if the normed linear space is finite dimensional then show that both the notions are one and the same.

Or

(b) Show that the null space of a closed linear operator $\tau : X \to Y$ between normed linear spaces is a closed subset of X.

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. Define l^p spaces for $1 \leq p \leq \infty$. Find out in detail the dual of l^p for $1 < p < \infty$.

17. Define C $[a, b]$. Show that it is not complete. Obtain its completion in detail.

18. Let H be Hilbert space. If H is separable then show that every other normal set is countable and that if H contains an orthonormal sequence which is total in H then H is separable.

19. State and prove Baire-Category theorem.

20. If $T : X \to Y$ is bounded linear operator between Banach spaces then show that image $T(B_0)$ of the open unit ball in X contains an open ball around $\Theta \in Y$.

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RW-6138	511303

M.Sc., DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** questions.

1. If γ is the curve $\bar{r} = \bar{r}\,(u)$ of class $\geq$ 1, then find the equation of the osculating plane.

2. Prove that a necessary and sufficient condition that a curve to be a straight line is that K = 0 at all points.

3. Define involute of c and also write the equation of the involute.

4. For the osculating sphere, write the radius of spherical curvature and the position vector of centre of curvature.

5. Define an anchor ring.

6. Define family of curves and prove that a first order differential equation of the form $P(u,v)\,du + Q(u,v)\,dv = 0$, where P and Q are C' functions which do not vanish simultaneously, is the curve $\phi = C$.

7. Write characteristic property of geodesic and also prove that at every point of a geodesic the rectifying plane is tangent to the surface.

8. Define geodesic curvature and prove that the geodesic curvature of a geodesic is zero.

9. Write second Fundamental form and also state the Meusnier's theorem.

10. Describe shortly about Dupins indicatrix.

Part - B (5 × 5 = 25)

Answer **all** questions.

11 (a) Obtain a formula for the arc length of a path given in terms of a parameter and also show that the length of one complete turn of the helix $r = (a\cos u, a\sin u, bu), a > 0, -\infty < u < \infty$ is 2π C, where $C = \sqrt{a^2 + b^2}$.

(*Or*)

(b) Define curvature and torsion. Show that $[\dot{r}, \ddot{r}, \dddot{r}] = 0$ is a necessary and sufficient condition for the curve to be a plane curve.

12. (a) If the radius of spherical curvature is constant, prove that the curve either lies on a sphere (or) has constant curvature.

(*Or*)

(b) Show that the torsion of an involute of a curve is equal to $\dfrac{\rho\,(\sigma\rho' - \sigma'\rho)}{(\rho^2 + c^2)\;(c - s)}$.

13 (a) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes z = const.

(*Or*)

(b) Prove that the metric is invariant under the parametric transformation.

14 (a) Prove that the curves of the family $\dfrac{v^3}{u^2}$ = constant are geodesics on s surface with metric $v^2 du^2 - 2uv\ du\ dv + 2\ u^2\ dv^2$ ($u > 0, v > 0$).

(*Or*)

(b) Prove that every helix on a cylinder is a geodesic.

15 (a) Prove that the Gaussian curvature of any developable is necessarily zero.

(*Or*)

(b) If there is a surface of minimum area passing through a closed space curve, prove that it is necessarily a surface of zero mean curvature.

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. Obtain the curvature and torsion of the curve of intersection of the two quadratic surfaces $ax^2 + by^2 + cz^2 = 1, a'x^2 + b'y^2 + c'z^2 = 1$.

17. State and prove the fundamental theorem for space curves.

18. If surfaces s and s' are isometric then prove that there exists a correspondence $u' = \phi(u,v), v' = \psi(u,v)$ between their parameters, where ϕ and ψ are single valued and have non-vanishing Jacobian such that the metric of s transforms into the metric of s'. Find a surface of revolution which is isometric with a region of the right helicoid.

19. Define geodesics and describe how to bind the equation for geodesics.

20. Prove that a necessary and sufficient condition that a curve on a surface be a line of curvature is that the surface normale along the curve form a developable.

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RW-6139

511503

M.Sc., DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

Elective : III—STOCHASTIC PROCESSES

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define Markov chain with an example.

2. Define the persistent state and transient state of Markov chain.

3. Write any two postulates for Poisson process.

4. Write the relation between the Poisson process and binomial distribution.

5. Define Wiener process and show that Wiener process is Gaussian.

6. Write down the forward and backward diffusion equations of Wiener process.

7. Define Gatton-Watson branching process with offspring distribution.

8. Define probability of Extinction.

9. Define inter arrival time and traffic intensity of a Queueing system.

10. Briefly explain the Queueing Model M/G/1.

Part - B (5 × 5 = 25)

Answer **all** questions choosing **either** (a) **or** (b).

11 (a) If state j is persistent then prove that as $n \to \infty,\ p_{jj}^{(nt)} \to t/v$.

(*Or*)

(b) If the state j is persistent, then prove that $F_{jk} = 1$ for every state K that can be reached from state j.

12. (a) Suppose that customers arrive at a Bank according to a Poisson process with a mean rate of a per minute. Then the number of customers N (t) arriving in an interval of duration t minutes follows a Poisson distribution with mean at. If the rate of arrival is 3 per minute, then in an arrival of 2 minutes, find the probability that the number of customers arriving is (i) exactly 4 (ii) less than 4.

(*Or*)

(b) Prove that the sum of two independent Poisson processes is a Poisson process.

13 (a) Derive forward diffusion equation of the Wiener process.

(*Or*)

(b) Let X (t) be a Wiener process with $\mu = 0$ and X (0) = 0. Find the distribution of T_{a+b}, for $0 < a < a + b$.

14 (a) If $m = E(X_1) = \sum_{k=0}^{\infty} kp_k$ and $\sigma^2 = \mathrm{Var}(X_1)$ then prove that $E\{X_n\} = m^n$, $\mathrm{Var}(X_n) = \frac{m^{n-1}(m^n - 1)}{m-1}\sigma^2$, if $m \neq 1$, $n\sigma^2$ if $m = 1$.

(*Or*)

(b) For a G.W. process with $m = 1$ and $\sigma^2 < \infty$, prove that $\frac{1}{n}\left\{\frac{1}{1-P_n(s)} - \frac{1}{1-s}\right\}$ converges to $\frac{\sigma^2}{2}$ uniformly in $0 \leq s < 1$.

15 (a) The arrivals at a counter in a bank occur in accordance with a Poisson process at an average rate of 8 per hour. The duration of service of a customer has exponential distribution with mean of 6 minutes. Find the probability that an arriving customer : (i) has to wait on arrival (ii) finds 4 customers in the queue.

(*Or*)

(b) Patients arrive at the outpatient department of a hospital in accordance with a Poisson process at the mean rate of 12 per hour, and the distribution of time for examination by an attending physician is exponential with a mean of 10 minutes. What is the minimum number of physicians to be posted for ensuring a steady state distribution ? For this number, find the expected waiting time of a patient prior to being examined.

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that limit $v_k = \underset{n \to \infty}{\text{Um}}\, p_{jk}^{(n)}$ for an irreducible ergodic chain exit and are independent of the initial state j.

17. Prove that under the postulates for Poisson process, N(t) follows Poisson distribution with mean λt.

18. Derive differential equation for a Wiener process.

19. Prove that the generating function $F(t,s)=\sum_{k=0}^{\infty} P_r\{X(t)=k\} s^k$ of an age-dependent branching process {X (t), t ≥ 0), $X_0 = 1$ satisfies the integral equation F (t, s)) = [1 – G (t)}s + $\int_0^t p(F(t-u,s))\, dG(u)$.

20. For the Queueing Model M/M/1 under steady state behaviour, find the waiting time in the Queue.

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RW-6140

511703

M.Sc., DEGREE EXAMINATION, NOVEMBER 2010

Maths

I/D : IMAGE PROCESSING AND PATTERN RECOGNITION

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** questions.

1. What is 'Image Acquisition' ?

2. Define : Heu, Saturation.

3. Name any two edge detection methods.

4. Write the mathematical form for opening and closing images.

5. What is meant by 'Redundancy' ?

6. What is 'Compression Ratio' ?

7. Write any two applications of Pattern Recognition.

8. What is meant by 'Pattern Classification' ?

9. What is Clustering ?

10. Define 'Single Prototype'.

Part - B (5 × 5 = 25)

Answer **all** questions.

11 (a) Explain the operation of video camera.

(*Or*)

(b) Explain and write mathematical forms for Convolution and Template.

12. (a) Explain Pyramid Edge Detection.

(*Or*)

(b) Explain Crack Edge Detection.

13 (a) Explain Quantizing Compression.

(*Or*)

(b) Write short notes on Image Standards.

14 (a) Explain Pattern Recognition Concept.

(*Or*)

(b) What is Generalized Decision Function ? Explain briefly.

15 (a) Write short notes on 'Cluster Seeking'.

(*Or*)

(b) Explain the use of Formal Language theory in Pattern Recognition.

Part - C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Explain Human Vision System and acquisition of images.

17. Explain in detail about Edge detection and morphological operations.

18. Describe design concepts and methodologies for pattern recognition.

19. Explain Fractal and Contour coding compression methods.

20. Explain in detail about syntactic pattern recognition.

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RW-6141 **511201**

M.Sc., DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

ALGEBRA—II

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** questions.

1. If S,T are subsets of V, then prove that L (SVT)= L (S) + L (T)

2. Prove that $|| \alpha_a || = | \alpha | \; || a ||$

3. Is every algebraic extension a finite extension ? Justify.

4. If F is a field of characteristic $p \neq 0$, prove that the derivative of the polynomial $x^p - 5$ is 0.

5. If G is a group of automorphism of the field K, prove that the fixed field of G is a subfield of K.

6. Is S_4 a Solvable group ? Justify.

7. Give an example of an element in A (0) which is not invertible but right invertible.

8. If V is finite dimension over F, then for S, $T \in A(V)$ prove that $r(ST) \leq r(T)$.

9. For A, B $\in F_n$, prove that $tr(AB) = tr(BA)$.

10. If $(vT, vT) = (v, v)$ for $v \in V$ then prove that T is unitary.

Part - B (5 × 5 = 25)

Answer **all** questions choosing **either** (a) **or** (b).

11 (a) If V is finite-dimensional and $\upsilon \neq 0 \in V$, then prove that there is element $f \in \hat{V}$ such that $f(\upsilon) \neq 0$.

(*Or*)

(b) Let V be a finite dimensional inner product space. Let W be a subspace of V, then prove that $V = W + W^{\perp}$. Prove also that V is the direct sum of W and $W^{\perp}$.

12. (a) Prove that the algebraic over algebraic is algebraic.

(*Or*)

(b) If $p(x)$ is a polynomial F $[x]$ of degree $n \geq 1$ and if $p(x)$ is irreducible over F, prove that there is an extension E of F such that $[E : F] = n$ in which $p(x)$ has a root.

13 (a) If K is a field and $\sigma_1, \sigma_2, \ldots, \sigma_n$ are distinct automorphism of K, then prove that it is impossible to find elements $a_1, a_2, \ldots, a_n$ not all 0, in K such that

$$a_1\sigma_1(u) + a_2\sigma_2(u) + \ldots + a_n\sigma_n(u) = 0$$

for all $u \in$ K.

(*Or*)

(b) Define a solvable group. If G is a group such that $G^{(k)} = e$ for some k, prove that G is solvable. Is the converse true ? Justify.

14 (a) If V is finite-dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V onto V.

(*Or*)

(b) If $T \in A(V)$ is nil potent then prove that $a_0 + a_1T + a_2T^2 + \ldots + a_nT^n$ where $a_i \in F$, is invertible if $a_0 \neq 0$.

15 (a) Define Hermitian transformation. If $T \in A(V)$ is Hermitian, then prove that all its characteristic roots are real.

(*Or*)

(b) If N is normal and AN = NA, then prove that $AN^* = N^*A$.

Part - C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Let V be a finite-dimensional inner product space, then prove that V has an orthonormal set as a basis. Illustrate this construction with an example.

17. Let F be a field and K be an extension of F. Show that an element $a \in K$ is algebraic over F if and only if F (a) is a finite extension of F.

18. If F is field of rationals, $W = e^{2\pi i/5}$ and K= F (w), then prove that G (K, F) is a cyclic group of order 4 and the fixed field of G (K, F) is F itself.

19. Prove that two nilpotent linear transformation are similar if and only if they have the same invariants.

20. Prove that a division ring is necessarily a commutative field.

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RW-6142

511202

M.Sc., DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

MEASURE AND INTEGRATION

(CBCS—2008 onwards)

Time : 3 Hours Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** questions.

1. Prove that outer measure is translation invariant.

2. Show that every σ-algebra is a σ-ring but the converse is not true.

3. Define Lebesgue measurable function.

4. Show that for any measurable function f, Ess sup $f = -$ ess inf $(-f)$.

5. Define Lebesgue integral.

6. Let f be bounded and let f and $|f|$ be Riemann integrable on $(-\infty, \infty)$. Then prove that f is integrable and $\int_{-\infty}^{\infty} f\,dx = \mathrm{R}\int_{-\infty}^{\infty} f\,dx$.

7. Define total variation of a signed measure.

8. If μ is a measure, $\int f\,d\mu$ exists and $\nu(\mathrm{E}) = \int_{\mathrm{E}} f\,d\mu$ prove that $\nu \le \mu$.

9. Define rectangle and measurable rectangle.

10. Let $[[\mathrm{X}, \delta, \mu]]$ and $[[\mathrm{Y}, \mathcal{T}, \nu]]$ be σ-finite measure spaces. Define the product space $\mu \times \nu$ on $\delta \times \mathcal{T}$.

Part - B (5 × 5 = 25)

Answer **all** questions.

11 (a) Prove that every interval is measurable.

(*Or*)

(b) Show that there exist uncountable sets of zero measure.

12. (a) Suppose that f is any extended real-valued function which for every x and y satisfies $f(x) + f(y) = f(x + y)$. Show that if f is measurable and finite then $f(x) = x f(1)$ for each x.

(*Or*)

(b) Prove that every measurable set need not be a Borel set.

13 (a) Let f and g be non-negative measurable functions prove that if a $\geq$ 0 then $\int af\ dx = a\int f\ dx$.

(*Or*)

(b) Show that $\int\limits_0^1 \frac{x^{1/3}}{1-x} \log\frac{1}{x}\, dx = 9\sum\limits_{n=1}^{\infty} \frac{1}{(3n+1)^2}$.

14 (a) Prove that a countable union of sets positive with respect to a signed measure ν is a positive set.

(*Or*)

(b) Let μ be a measure and let the measure ν_1, ν_2 be given by $\nu_1(\mathrm{E}) = \mu(\mathrm{A} \cap \mathrm{E}), \nu_2(\mathrm{E}) = \mu(\mathrm{B} \cap \mathrm{E})$, where $\mu(\mathrm{A} \cap \mathrm{B}) = 0$ and E, A, B $\in \delta$. Show that $\nu_1 \perp \nu_2$.

15 (a) Prove that $\mathcal{S} \times \mathcal{T} = \mathcal{M}_0(\mathcal{E})$.

(*Or*)

(b) State an prove Fubini's theorem.

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that the class $\mathcal{M}$ is a σ-algebra.

17. Let c be any real number and let f and g be real-valued measurable functions defined on the same measurable set E. Then prove that $f + c$, cf, $f + g$, $f - g$ and fg are measurable

18. State and prove Fatou's Lemma.

19. Let ν be a signed measure $[[X, \delta]]$. Let $E \in \delta$ and ν (E) > 0. Then prove that there exist A, a set positive with respective to ν, such that $A \subseteq E$ and ν (A) > 0.

20. Let $[[X, \delta, \mu]]$ and $[[Y, \mathcal{J}, \nu]]$ be σ-finite measure spaces.For $V \in \delta \times \mathcal{J}$ write

$\phi(x) = \nu(V_x), \psi(y) = \mu(V^y)$, for each $x \in X, y \in Y$.

Prove that $\int_X d\mu(x) \int_Y \chi_V(x,y) d\nu(y) = \int_Y d\nu(y) \int_X \chi_V(x,y)\, d\mu(x)$.

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RW-6143

511203

M.Sc., DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

TOPOLOGY

(CBCS—2008 onwards)

Time : 3 Hours | Maximum : 75 Marks

Part - A (10 × 2 = 20)

Answer **all** the questions.

1. Show that if G is a basis for a topology on X then the topology generated by G equals the intersection of all topologies on X that contain G.

2. Show that product of two Hausdorff spaces is Hausdorff.

3. Show that every open ball in R^n is path connected.

4. Give an example of a connected space which is not locally connected.

5. Prove that every closed interval in R is uncountable.

6. Show by an example that completeness is not a topological property.

7. Prove that a subspace of a regular space is regular.

8. Show that if X is normal, every pair of disjoint closed sets have neighbourhoods whose closures are disjoint.

9. Let X be a space. Let $\mathscr{D}$ be a maximal collection of subsets of X satisfying the finite intersection condition. Let $D \in \mathscr{D}$. Show that if $D \subset A$, then $A \in \mathscr{D}$.

10. Define a compactification of a space X, and give a compatification of $(0,1)$.

Part - B (5 × 5 = 25)

Answer **all** questions.

11 (a) Define the lower limit topology on R. Prove that the lower limit topology on R is strictly finer than the standard topology on R.

(*Or*)

(b) Let A be a subset of a Hausdorff space X. Prove that every finite set in x is closed and x is a limit point of A, if and only if every neighbourhood of x contains infinitely many points of A.

12. (a) Prove that the union of a collection of connected sets that have a point in common is connected.

(*Or*)

(b) Prove that the path components of a space X are path-connected disjoint subsets of X whose union is X, such that each path-connected subset of X intersects only one of them.

13 (a) Prove that a closed subset of a compact space is compact. Also prove that every compact subset of a Hausdorff space is closed.

(*Or*)

(b) Let X be a locally compact Hausdorff space which is not compact and let Y be the one-point compactification of X. The prove that Y is a compact Hausdorff space and X is a subspace of Y and $\bar{X} = Y$.

14 (a) Prove that a space satisfying the second countability axiom is both Lindelöff and separable.

(*Or*)

(b) Prove that every well-ordered set X is normal in the order topology.

15 (a) Prove that a subspace of a completely regular space is completely regular. Also prove that a product of completely regular spaces is completely regular.

(*Or*)

(b) Let X be a space. Then prove that the following are equivalent :

(i) X is completely regular.

(ii) X is homoeomorphic to a subspace of a compact Hausdorff space.

(iii) X is homoeomorphic to a subspace of a normal space.

Part - C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Let X be a metrizable space and Y be a topological space. Then prove that a function $f : X \to Y$ is continuous if and only if for every convergent sequence $x_n \to x$ in X, the sequence $f(x_n)$ converges to $f(x)$.

17. If L is a linear continuum in the order topology then prove that L is connected and so is every interval and ray in L.

18. State and prove the Ascoli's theorem, classical version.

19. State and prove the Urysohn lemma.

20. State and prove the extension property of the Stone-Cechcompactification of a completely regular space. Also prove that Stone-Cech compactification is essentially unique and is characterized by the extension property.

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RW-6144 | **511402**

M.Sc., DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

COMPLEX ANALYSIS

(CBCS—2008 onwards)

Time : 3 Hours | Maximum : 75 Marks

Part - A $(10 \times 2 = 20)$

Answer **all** questions.

1. Find the value of $\tan(1 + i)$.

2. Verify C.R. equations for the function z^3.

3. Evaluate $\int_c x\,dz$ where C is the line segment from 0 to $1 + i$.

4. Evaluate $\int_C \frac{z\,dz}{z^2 - 1}$, where C is the positively oriented circle $|z| = 2$.

5. Evaluate $\int_{0}^{2\pi} \frac{d\theta}{5+4\sin\theta}$.

6. How many roots does the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ have in the disk $|z| < 1$?

7. Define a harmonic function and give an example.

8. State Maximum principle for harmonic functions.

9. Define genus of a canonical product.

10. State Legendre's duplication formula.

Part - B (5 × 5 = 25)

Answer **all** questions.

11 (a) State and prove Luca's theorem..

(*Or*)

(b) State and prove Abel's limit theorem.

12. (a) Prove that an analytic function comes arbitrarily close to any complex value in every neighbourhood of an essential singularity.

(*Or*)

(b) Suppose that $\phi(t)$ is continuous on the arc v.

Prove that the function $F_n(z) = \int_v \frac{\phi(t)dt}{(t-z)^n}$ is analytic in each of the regions determined by v and its derivatives is $F_n'(z) = nF_{n+1}(z)$.

13 (a) State and prove Residue theorem.

(*Or*)

(b) State and prove Rouche's theorem.

14 (a) State and prove Poisson's formula.

(*Or*)

(b) State and prove Hurwitz theorem.

15 (a) Prove that a necessary and sufficient condition for the absolute convergence of the product $\prod_{n=1}^{\infty}(1+a_n)$ is the convergence of the series $\sum_{n=1}^{\infty}|a_n|$.

(*Or*)

(b) Prove that :

$$(2\pi)^{\frac{n-1}{2}}\Gamma(z)=n^{z-\frac{1}{2}}\Gamma\left(\frac{z}{n}\right)\cdot\Gamma\left(\frac{z+1}{n}\right)\ldots\Gamma\left(\frac{z+n-1}{n}\right)$$

Part - C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Find the analytic function $f(z)\ u+iv$, if

$u+v=\dfrac{\sin 2x}{\cosh 2y-\cos 2x}$.

(b) Find the bilinear transformation which carries $0, i, -i$ into $1, -1, 0$.

17. State and prove Cauchy's theorem for a rectangle.

18. Evaluate $\int_0^{\infty} \frac{\log x \, dx}{(1+x^2)}$ by the method of residues.

19. (a) Develop $\frac{1}{(1+z^2)}$ in powers of $z-1$.

(b) Prove that $\sum_{n=1}^{\infty} \frac{nz^n}{1-z^n} = \sum_{n=1}^{\infty} \frac{z^n}{(1-z^n)^2}$ for $|z| < 1$.

20. State and prove Weirstrass theorem on the existence of entire function with arbitrary prescribed zeros.

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RW-6145 **511101**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

ALGEBRA—I

(CBCS–2004 onwards)

Time : 3 Hours Maximum : 100 Marks

Answer **all** questions. (5 × 20 = 100)

Each question carries 20 marks.

1. (a) (i) Let H and K be finite subgroups of G of orders o(H) and o(K) respectively. Prove that $o(HK) = \dfrac{o(H) \cdot o(K)}{o(H \cap K)}$.

(ii) Let ϕ be a homomorphism of a group G onto a group $\overline{G}$ with kernel K. Prove that K is a normal subgroup of G. Also prove that G/K is isomorphic to $\overline{G}$.

(*Or*)

(b) (i) Let G be a group, H, a subgroup of G and S be the set of all right cosets of H in G. Prove that there is a homomorphism θ of G into A(S) and the kernel of θ is the largest normal subgroup of G which is contained in H.

(ii) Let I(G) be the group of inner automorphisms of G and let Z be the centre of G. Prove that I(G) is isomorphic onto G/Z.

2. (a) (i) State and prove Cauchy's theorem for a finite group.

(ii) State and prove the third part of Sylow's theorem.

(*Or*)

(b) (i) Let G be a group and suppose that G is the internal direct product of N_1, $N_2, \ldots, N_k$. Let $T = N_1 \times N_2 \times \ldots \times N_k$. Prove that G and T are isomorphic.

(ii) State and prove the second part of Sylow's theorem.

3. (a) (i) Let R be a commutative ring with unit element. Prove that R is a field if, and only if, its only ideals are {0} and R only.

(ii) State and prove Fermat's theorem.

(*Or*)

(b) (i) If V and W are vector spaces over a field F of dimensions m and n respectively prove that the dimension of the vector space Hom (v,w) over F is $m.n$.

(ii) State and prove Schwarz inequality.

4. (a) (i) If $p(x)$ is a polynomial in F(x) of degree $n \geq 1$ and is irreducible over F, prove that there is an extension E of F such that [E : F] = n, in which $p(x)$ has a root.

(ii) If F is of characteristic zero and if a,b are algebraic over F, prove that there exists an element $C \in F(a,b)$ such that $F(a,b) = F(c)$.

(*Or*)

(b) (i) For every prime number p and every positive integer m prove that there is a unique field having p^m elements.

(ii) Prove that K is a normal extension of F if, and only if, K is the splitting field of some polynomial over F.

5. (a) (i) If V is finite dimensional over F prove that $T \in A(V)$ is regular if, and only if, T maps V onto V.

(ii) If $\lambda \in F$ is a characteristic root of $(T \in A(V)$ prove that for any polynominal $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of $q(T)$. Deduce that λ is a root of minimal polynomial for T and T has a finite number of characteristic roots in F.

(*Or*)

(b) (i) If $T \in A(V)$ has all its characteristic roots in F, prove that there is a basis of V in which the matrix of T is triangular.

(ii) If $T \in A(V)$ is nilpotent and T is of index of nilpotent n_1 prove that a basis of V can be found such that the matrix of T in this basis has the form.

$$\begin{pmatrix} M_{n_1} & 0 & \cdots & 0 \\ 0 & M_{n_2} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & M_{n_r} \end{pmatrix}$$

where

$n_1 \geq n_2 \geq \ldots \geq n_r$ and $n_1 + n_2 + \ldots + n_r = \dim_F V$.

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M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

REAL ANALYSIS

(CBCS–2004 onwards)

Time : 3 Hours Maximum : 100 Marks

Answer **all** questions. (5 × 20 = 100)

Each questions carries 20 marks.

1. (a) (i) If $k \geq 2, x \in \mathbb{R}^k$ then prove that there exists $y \in \mathbb{R}^k$ such that $y \neq 0$ but $x \cdot y = 0$.

(4)

(ii) Show that the set of all rational numbers form a field. (6)

(iii) Define the space $\mathbb{R}^k$ $(k = 1, 2...)$. What is $|x|$ for $x \in \mathbb{R}^k$. Prove in all details that $\mathbb{R}^k$ is a metric space.

(10)

(*Or*)

(b) (i) If x and y are complex then show that $\big||x| - |y|\big| \leq |x - y|$

(4)

(ii) State and prove Schwarz inequality for complex numbers.

(6)

(iii) Show that every positive number x has a unique positive nth root denoted by $x^{1/n}$ and that $(ab)^{1/n} = a^{1/n}\, b^{1/n}$ for $a, b > 0$.

(10)

2. (a) (i) Define continuity at a point for a map between two metric spaces. Show that composition of continuous maps is continuous whenever the composition exists.

(4)

(ii) If $f : X \to Y$ is a map between metric spaces then show that f is continuous if and only if $f^{-1}(V)$ is open for every V open in Y.

(6)

(iii) Define component function of $f : X \to \mathbb{R}^k$ where X is a metric space. Show that f is continuous if and only if its components are continuous. Show also that if f, g are two continuous maps from X into $\mathbb{R}^k$ then so are $(f + g)$ and $f \cdot g$ where $\cdot$ denotes the inner product.

(10)

(*Or*)

(b) (i) Define uniform continuity of a map $f : X \to Y$ where X and Yare metric spaces and show that if f is continuous and X is compact then f is uniformly continuous.

(10)

(ii) Let E be non-compact in $\mathbb{R}$. Show that there is a continuous function on E which is not bounded and that there is a continuous bounded function on E having no maximum. If in addition E is bounded then show that there is a continuous function on E which is not uniformly continuous.

(10)

3. (a) (i) Show that the functions $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ defined by $f(x) = |x|$ and $g(x) = \sin {}^{1}\!/_{x}$ for $x \neq 0$ and $g(0) = 0$ are both continuous at all points in $\mathbb{R}\setminus\{0\}$ and that f is continuous at $x = 0$ but not g. Show also that f is not differentiable at $x = 0$.

(10)

(ii) Show that if $f : [a, b] \to \mathbb{R}$ is such that f' is continuous then given $\varepsilon > 0$ we can find a $\delta > 0$ such that for

$$t_2\ x \in [a,b],\ 0 < |t - x| < \delta \Rightarrow \left| \frac{f(t) - f(x)}{t - x} - f'(x) \right| < \varepsilon.$$

(10)

(*Or*)

(b) (i) State and prove Taylor's theorem in all details after proving Mean Value Theorem for real differentiable functions.

(10)

(ii) Show that if $f : [a,b] \to \mathbb{R}$ is differentiable and has a local maximum at $x \in (a,b)$ then $f'(x) = 0.$ Use this to show that if $f'(a) < \lambda < f'(b)$ then there exists a point $x \in (a,b)$ such that $f'(x) = \lambda.$ Deduce that f can not have simple discontinuities on $[a,b]$.

(10)

4. (a) (i) For a bounded function f on $[a,b]$ define $L(P, f, \alpha)$ and $U(P, f, \alpha)$ where P is a partition of $[a,b]$ and α is an increasing function. Define upper and lower integrals of f with respect to α. Show that $L(P_1, f, \alpha) \leq U(P_2, f, \alpha)$ for any two partitions P_1 and P_2. How are the upper sums and lower sums related for partitions P and P* if P* is a refinement of P ? Deduce $\int_{-a}^{b} f d\alpha \leq \int_{a}^{-b} f d\alpha.$

(10)

(ii) Show that if f is bounded and has finitely many points of discontinuity on [a,b] at each of which α is continuous, the $f \in R(\alpha)$ on $[a,b]$. Show also that if f is monotonic, α is continuous and increasing then $f \in R(\alpha)$.

(10)

(*Or*)

(b) (i) Assume α is increasing and $\alpha' \in R$ on $[a,b]$ .Let $f:[a,b] \to \mathbb{R}$ be bounded. Show that $f \in R(\alpha) \Leftrightarrow f\alpha' \in R$ and that in this case $\int_a^b f d\alpha \leq \int_a^b f\alpha' dx$.

(10)

(ii) Show that every continuous Riemann Integrable function f on $[a, b]$ has an anti-derivative. i.e., there exists g on $[a,b]$ such that $g'(x) = f(x)$.

(6)

(iii) State and prove the fundamental theorem of Integral Calculus.

(4)

5. (a) (i) Define an equi-continuous family of functions on a set E and give an example. Show that every sequence of pointwise bounded complex functions on a countable set E admits a convergent subsequence (pointwise) on E.

(10)

(ii) Show that if $f_n \in \mathscr{C}(k)$, k-compact and $\{f_n\}$ converges uniformly on k then $\{f_n\}$ equicontinuous on k.

(6)

(iii) Give an example of a sequence $\{f_n\}$ defined on [0,1] such that $\{f_n\}$ is uniformly bounded on [0,1] pointwise limit of $\{f_n(x)\}$ exists but admits no subsequence converging uniformly on [0,1].

(4)

(*Or*)

(b) (i) State and prove Stone-Weierstrass theorem in all details.

(16)

(ii) For what values of x does the series $\sum_{n=1}^{\infty} \frac{1}{1+n^2x}$ converge absolutely ? Why ? Does there exist a closed interval $[c, d]$ in which it is uniformly convergent ? Justify.

(4)

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RW-6148 **511501**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

ANALYTIC NUMBER THEORY

(CBCS—2004 onwards)

Time : 3 Hours Maximum : 100 Marks

Answer **all** questions. (5 × 20 = 100)

1\. (a) (i) Given any two integers a, b show that there exist is a common divisor d of a and b of the form $d = ax + by$ where x, y are integers.

(ii) State and prove division algorithm.

(*Or*)

(b) (i) Show that if $n \geq 1$ then $\psi(n) = n \prod_{p/n} \left(1 - \frac{1}{p}\right)$.

(ii) State and prove Möbius inversion formula.

2. (a) State and prove Dirichlet's asymptotic formula.

(*Or*)

(b) Show that the set of lattice points visible from the origin has density $6/\Pi^2$.

3. (a) Show that the following are equivalent

$$\lim_{x\to\infty} \frac{\Pi(x)\log x}{x} = 1$$

$$\lim_{x\to\infty} \frac{S(x)}{x} = 1$$

$$\lim_{x\to\infty} \frac{\psi(x)}{x} = 1$$

(*Or*)

(b) State and prove Tauberian theorem.

4. (a) (i) Show that if a prime p does not divide a then $a^{p-1} \equiv 1 \pmod{p}$.

(ii) State and prove Lagrange's theorem.

(*Or*)

(b) (i) State and prove Wilson's theorem.

(ii) Show that the set of lattice points in the plane visible from the origin contains arbitrary large square gaps.

5. (a) (i) State and prove Euler's criterion.

(ii) State and prove Gauss Lemma.

(*Or*)

(b) State and prove Reciprocity law for Jacobi symbols.

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RW-6149 | **511701**

M.Sc., DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

OBJECT ORIENTED PROGRAMMING AND C++

(CBCS–2004 onwards)

Time : 3 Hours Maximum : 100 Marks

Answer **all** questions. $(5 \times 20 = 100)$

All questions carry equal marks.

1. (a) Explain Basic concepts and Applications of OOPs Programming ?

(*Or*)

(b) Write the structure of a C++ program and Explain with Suitable example ?

2. (a) Write a C++ program to find the volume of various shapes using function overloading ?

(*Or*)

(b) Explain template functions with a program ? List its Merits and demerits.

3. (a) Explain Multiple constructors in a class with an example ?

(*Or*)

(b) Explain the various rules for constructors and destructors ?

4. (a) Write a C++ program change the sign of given values using overloading Unary minus operators ?

(*Or*)

(b) Explain about Operator overloading concepts and its Rules ?

5. (a) Explain about the concepts of Inheritance and its different types ?

(*Or*)

(b) Explain polymorphism with virtual functions with an example ?

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RW-6150 **511201**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

ALGEBRA—II

(CBCS–2004 onwards)

Time : 3 Hours Maximum : 100 Marks

Answer **all** questions. $(5 \times 20 = 100)$

Each question carries 20 marks.

1. (a) (i) Let V be a finite dimensional inner product space. Prove that V has an orthonormal set as a basis.

(ii) Let V be a vector space over F. If A and B are subspaces of V, prove that $\frac{A+B}{B}$ is isomorphic to $\frac{A}{A \cap B}$.

(*Or*)

(b) (i) If V is a finite–dimensional vector space over F and if W is a subspace of V, prove that W is finite–dimensional, $\dim W \leq \dim V$ and $\dim \frac{V}{W} = \dim V - \dim W$.

(ii) If V is finite–dimensional and $v\,(\neq 0) \in V$. Prove that there is an element $f \in \hat{V}$ such that $f(v) \neq 0$.

2. (a) (i) Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a non-trivial common factor.

(ii) If L is a finite extension of the field K and if K is a finite extension of the field F, prove that L is a finite extension of F and $[L:F] = [L:K]\;[K:F]$.

(*Or*)

(b) (i) Prove that a polynomial of degree n over a field can have atmost n roots in any extension field.

(ii) If F is a field of characteristic 0 anf if a,b are algebraic over F, prove that there is an element $C \in F(a,b)$ such that $F(a,b) = F(c)$.

3. (a) (i) Prove that K is the normal extension of the field F if and only if K is the splitting field of some polynomial over F.

(ii) If K is a field and if $\sigma_1, \sigma_2 ..., \sigma_n$ are distinct automorphisms of K, prove that it is impossible to find elements $a_1, a_2 ..., a_n$ not all zero in K such that $a_1, \sigma_1(u) + a_2 \sigma_2(u) + ... + a_n \sigma_n(u) = 0$ for all $u \in K$.

(*Or*)

(b) (i) Prove that a group G is solvable if and only if $G^{(k)} = (e)$ for some integer K.

(ii) Prove that the polynomial $p(x) = x^5 - 6x + 3$ over Q is irreducible and have exactly two nonreal roots.

4. (a) (i) If V is finite–dimensional over F, prove that $T \in A(V)$ is regular if and only if the constant term of the minimal polynomial for T is not 0.

(ii) If V is n–dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then T satisfies a polynomial of degree n over F.

(*Or*)

(b) If $T \in A(V)$ is nilpotent, of index of nilpotence n_1, prove that a basic of V can be found such that the matrix of T in this basis has the form.

$$\begin{pmatrix} M_{n_1} & 0 & \dots & 0 \\ 0 & M_{n_2} & \dots & 0 \\ 0 & 0 & \dots & M_{n_r} \end{pmatrix}$$

where $n_1 \geq n_2 \geq \dots \geq n_r$ and where $n_1 + n_2 + \dots + n_r = \dim V$.

5. (a) (i) If F is of characteristic 0 anf if S and T, in $A_F(V)$, are such that ST–TS commutes with S, then ST–TS is nilpotent. (Prove !)

(ii) For every prime number p and every positive integer m there is a unique field having p^m elements. (Prove !)

(*Or*)

(b) (i) Prove that the Hermitian linear transformation T is non-negative if and only if all of its characteristic roots are non-negative.

(ii) If F is a finite field and $\alpha \neq 0$, $\beta \neq 0$ are two elements of F, prove that there exist elements a and b in F such that $1 + \alpha a^2 + \beta b^2 = 0$.

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RW-6151 **511202**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

REAL ANALYSIS—II

(CBCS—2004 onwards)

Time : 3 Hours Maximum : 100 Marks

($5 \times 20 = 100$)

Answer **all** questions.
Each question carries 20 marks.

1. (a) (i) Let $\Sigma\, a_n$ be a series of complex terms whose partial sums form a bonded sequence. Let $\{b_n\}$ be a decreasing sequence which converges to zero. Prove that $\Sigma\, a_n\, b_n$ converges.

(5)

(ii) Let $\Sigma\, a_n$ be an absolutely convergent series having sum s. Show that every rearrangement of $\Sigma\, a_n$ also converges absolutely and has sum s.

(15 marks)

Or

(b) (i) Assume that $\sum_{n=0}^{\infty} a_n$ converges absolutely and has sum A, and suppose $\sum_{n=0}^{\infty} b_n$ converges with sum B. Prove that the Cauchy product of these two series converges and has sum AB.

(12)

(ii) If each $a_n > 0$, show that $\Pi\ (1 + a_n)$ converges if and only if the series $\Sigma\, a_n$ converges.

(8)

2. (a) (i) State and prove Cauchy condition for uniform convergence of a sequence of functions defines on a set S.

(ii) Let $f_n(x) = \dfrac{1}{nx+1}$ if $0 < x < 1$, $n = 1, 2, 3, \ldots..$ Show that $\{f_n\}$ converges pointwise but not uniformly on (0, 1).

(5)

(iii) Let $g_n(x) = \dfrac{x}{n\,x+1}$ if $0 < x < 1$, $n = 1, 2, 3, \ldots$. Prove that $g_n \to 0$ uniformly on $(0, 1)$.

(5)

Or

(b) Assume that each term of $\{f_n\}$ is a real valued function having a finite derivative at each point of an open interval (a, b). Assume that for at least one point x_o in (a, b) the sequence $\{f_n(x_0)\}$ converges. Assume further that there exists a function g such that $f_n^1 \to g$ uniformly on (a, b). Show that :

(1) There exists a function f such that $f_n \to f$ uniformly on (a, b).

(2) For each x in (a, b) the derivative $f'(x)$ exists and equals $g(x)$.

(20)

3. (a) (i) Let S be an open connected subset of R^h and let $f : S \to R^m$ be differentiable at each point of S. If $f'(C) = 0$ for each C in S, prove that f is constant on S.

(8)

(ii) If both partial derivatives $D_r f$ and $D_k f$ exist in an n–ball B (C ; δ) and if both are differentiable at C, prove that

$D_{r,k} f(C) = D_{k,r} f(C)$

(12)

Or

(b) (i) If $f = u + i\,v$ is a complex-valued function with a derivative at a point z in C, prove that $J_f(z) = |f'(z)|^2$.

(5)

(ii) State and prove the second-derivative test for extrema of vector valued functions.

(15)

4. (a) (i) Show that the outer measure of an interval is its length.

(15)

(ii) If E_1 and E_2 are measurable sets prove that $E_1 \cup E_2$ is also measurable.

(5)

Or

(b) Let E be a given set. Prove that the following five statements are equivalent :

(i) E is measurable.

(ii) Given $\varepsilon > 0$, there is an open set $O \supset E$ with $m^* (O \sim E) < \varepsilon$.

(iii) Given $\varepsilon > 0$, there is a closed set $F \subset E$, with $m^* (E \sim F) < \varepsilon$.

(iv) There is a G in G_δ with $E \subset G$, $m^* (G \sim E) = 0$.

(v) There is an F in $F\sigma$ with $F \subset E$, $m^* (E \sim F) = 0$.

5 (a) (i) Let f be defined and bounded on a measurable set E with ME finite. Prove that

$$\inf_{f \leq \psi} \int_E \psi(x)\, dx = \sup_{f \geq \phi} \int_E \phi(x)\, dx$$

for all simple functions ϕ and ψ if and only if f is measurable.

(15)

(ii) If f and g are non negative measurable functions defined on a measurable set E, prove that $\int_E (f+g) = \int_E f + \int_E g$.

(5)

Or

(b) (i) "State and prove : Bounded convergence theorem".

(8)

(ii) State and prove "Fatou's lemma".

(7)

(iii) State and prove "Monotone convergence theorem".

(5)

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RW-6152 **511203**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

TOPOLOGY

(CBCS–2004 onwards)

Time : 3 Hours Maximum : 100 Marks

Answer **all** questions. (5 × 20 = 100)

Each question carries 20 marks.

1. (a) (i) If $\mathcal{B}$ is a basis for a topology on a set X then define the topology $\mathfrak{J}$ generated by $\mathcal{B}$ and show that $\mathfrak{J}$ is infact a topology on X.

(6)

(ii) Let A be a subset of the topolgical space X and let A' be the set of all limit points of A. Then prove that $\overline{A} = A \cup A'$.

(6)

(iii) Let X and Y be topological spaces and let $f : X \to Y$. Then prove that the following are equivalent.

(1) f is continuous.

(2) For every subset A of X, $f(\overline{A}) \subset \overline{f(A)}$.

(3) For every closed subset B of Y, the set $f^{-1}(B)$ is closed in X.

(8)

(*Or*)

(b) (i) Let A, B be subsets of a topological space X; and let A' denote the set of all limit points of A. Determine whether the following equations hold; if an equality fails determine whether one of the inclusions $\supset$ or $\subset$ holds.

(1) $(A \cup B)' = A' \cup B'$.

(2) $(A \cap B)' = A' \cap B'$.

(9)

(ii) If d is a metric on the set X then show that the collection of all E–balls $B_d(x, \in)$, for $(x \in X)$ and $\in > 0$ is a basis for a topology on X. Also define the standard bounded metric $\bar{d}$ on X corresponding to d and prove that $\bar{d}$ is a metric on X and that d and $\bar{d}$ induce the same topology.

(11)

2. (a) (i) Let $\{X_\alpha\}_{\alpha \in J}$ be an indexed family of connected spaces. Prove that $\prod_{\alpha \in J} X_\alpha$ is connected in the product topology.

(15)

(ii) Prove that the components of X are connected disjoint subsets of X whose union is X, such that each connected subset of X intersects only one of them.

(5)

(*Or*)

(b) (i) Let $\{A_\alpha\}$ be a collection of connected subsets of X and let A be a connected subset of X. Show that if $A \cap A_\alpha \neq \Phi$ for all α, then $A \cup (\mathrm{U}\, A_\alpha)$ is connected.

(6)

(ii) State and prove the intermediate value theorem.

(6)

(iii) Prove that the relation $x \sim y$ if there is a path in X from x to y is an equivalence relation and each path connected subset of X intersects only one of the equivalence classes.

(8)

3. (a) (i) Prove that every compact subset of a Hausdorff space is closed.

(5)

(ii) Let X be a simply ordered set having the least upper bound property. Prove that in the order topology each closed interval in X is compact.

(15)

(*Or*)

(b) (i) State and prove the Lebesgue number lemma.

(10)

(ii) Let X be a compact space and (Y, d) be a compact metric space. Let $\mathscr{F}$ be a subset of C (X,Y). Then prove that $\mathscr{F}$ is equicontinuous if and only if $\mathscr{F}$ is totally bounded in the sup metric ρ.

(10)

4. (a) (i) Prove that every second countable space is a Lindelöf space and the converse is true if the space is metrizable.

(10)

(ii) Prove that every regular space with a countable basis is normal.

(10)

(*Or*)

(b) (i) Prove that the space S_Ω satisfies the first countability axiom but does not satisfy the second countability axiom.

(6)

(ii) Show by an example that every Hausdorff space is not regular.

(8)

(iii) State the Urysohn Lemma, Tietze extension theorem and the Urysohn metrization theorem.

(6)

5. (a) (i) Let $\mathscr{D}$ be a collection of subsets of a set X that is maximal with respect to f.i.c. Then prove that any finite intersection of elements of $\mathscr{D}$ belongs to $\mathscr{D}$ and if A is any subset of X that intersects every element of $\mathscr{D}$ then A belongs to $\mathscr{D}$.

(6)

(ii) Prove that any completely regular space is homeomorphic to a subspace of a compact Hausdorff space.

(4)

(iii) Define the Stone-Cech compactification $\beta(X)$ of a completely regular space X. And prove that every continuous real-valued function on X can be uniquely extended to a continuous real-valued function on $\beta(X)$.

(10)

(*Or*)

(b) State the prove the Tychonoff theorem.

(20)

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RW-6153 **511502**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

NUMERICAL ANALYSIS

(CBCS–2004 onwards)

Time : 3 Hours Maximum : 100 Marks

Answer **all** the questions.

($5 \times 20 = 100$)

1. (a) Using Steepest method, find the maxima and minima of the function $x_1^3/3 + x_2^2 \cdot x_1 + 3$.

(b) Write an algorithm for Gauss-Seidel iteration.

(*Or*)

(c) Write an algorithm for damped Newton's method.

(5)

(d) Solve the system $x^2 + xy^3 = 9$ and $3x^2 y - y^3 = 4$ by fixed point interation.

(15)

2. (a) Explain uniform approximation by polynomials.

(5)

(b) Solve the least squares approximation problem of $f_n \approx f(x_n)$ with $x_n = 10 + (n-1)|5;$ $n = 1, 2, ..., 6$ using orthogonal polynomials.

(15)

(*Or*)

(c) Derive Legendre polynomial $P_{k+1}(x)$.

(d) Find the Zeros of the Hermite polynomial $H_4(x)$.

3. (a) Derive Simpson's rule in numerical differentiation.

(b) Use Simpson's rule to estimate the value of the integral $\int_0^1 e^{-x^2} \cdot dx$.

(*Or*)

(c) Write a program to evaluate the integral $\int_1^3 (\sin x)^2 / x \cdot dx$. Using Gaussian quadrature with K = 3 and N = 2 subdivisions of the interval [1, 3].

(d) Explain Gaussian rules in numerical integration.

4. (a) Solve : $y''' - 2y'' - y' + 2y = x;\quad y(0) = 1;$ $y'(0) = -1;\quad y''(0) = 0.$

(b) Using Runge-Kutta method of fourth order, find y (0.2) and y (0.4) from $y'(x) = x^2 + y^2;\ y(0) = 0.$

(*Or*)

(c) Using Taylor series, find the solutions of the equation $xy' = x - y;\ y(2) = 2$ at $x = 2.1.$

(d) Using Euler's method solve $y' = x + y,\ y(0) = 1$ for x = 0.2, 0.4, 0.6, 0.8, and 1. Verify with exact solutions.

5. (a) Derive formula for Adams-Bashforth method.

(b) By the shooting method, solve $xy'' + 1 + (y')^2 = 0;\ y(0) = 1;\ y(1) = 2.$

(*Or*)

(c) By predictor corrector method, solve $y' = x - \frac{1}{y},\ y(0) = 1$ from $x = 0$ to $x = 0.2$ with $h = 0.1$.

(d) Explain finite-difference method to solve an ordinary differential equation, with an example.

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RW-6154	511302

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

FUNCTIONAL ANALYSIS

(CBCS—2004 onwards)

Time : 3 Hours — Maximum : 100 Marks

$(5 \times 20 = 100)$

Answer **all** questions.

Each question carries 20 marks.

1. (a) (i) $\{x_1, x_2, ..., x_n\}$ is a linearly independent set of vectors in a normed space X prove that for every choice of scalars $\alpha_1, \alpha_1\ ..., \alpha_n$ there is a number $c > 0$ such that $\|\alpha_1\, x_1 + ... + \alpha_n\, x_n\| \geq c\left(|\alpha_1| + |\alpha_2| + ... + |\alpha_n|\right)$.

(ii) If T is a linear Operator prove that :

1 The range R (T) is a vector space.

2 If dim D(T) = $n < \infty$ then dim R (T) $\leq n$.

3 The null space N (T) is a vector space.

(*Or*)

(b) (i) If Y is a Banach space, prove that B (X, Y) is a Banach space.

(ii) Prove that the dual space of l^1 is l^∞.

2. (a) Let X be an inner product space and M be a non empty convex subset which is complete. Prove that for every given $x \in X$ such that

$$\delta = \inf_{\bar{y} \in M} \|x - \bar{y}\| = \|x - y\|.$$

If M is a complete subspace Y and $x \in X$ is fixed prove that $z = x - y$ is orthogonal to Y.

(*Or*)

(b) (i) Prove that if a Hilbert space H contains an orthonormal sequence which is total in H then H is separable.

(ii) Prove that two Hilbert spaces H and $\tilde{H}$, both real or both complex are isomorphic iff they have the same Hilbert dimension.

3. (a) (i) Prove that every bounded linear functional f on a Hilbert space H can be represented as $f(x) = \langle x, z \rangle$ where z depends on f, is uniquely determined by f and has norm $\|z\| = \|f\|$.

(ii) If $\langle V_1, W \rangle = \langle V_2, W \rangle$ for all W in an inner product space X then prove that $V_1 = V_2$.

(*Or*)

(b) (i) Let $T : H \to H$ be a bounded linear operator on a Hilbert space H. Then prove that if T is self adjoint, $\langle Tx, x \rangle$ is real for $x \in H$ and prove that if H is complex and $\langle Tx, x \rangle$ is real for all $x \in H$, the operator T is self adjoint.

(ii) Let $T : H_1 \to H_2$ be a bounded linear operator where H_1 and H_2 are Hildert spaces. Prove that there exists a unique bounded linar operator $T^* : H_2 \to H_1$ such that $\langle Tx, y \rangle$ = $\langle x, T^* y \rangle$ for all $x \in H_1$ and $y \in H_2$ with $\|T^*\| = \|T\|$.

4. (a) State and prove generalised Hahn - Banach theorem.

(*Or*)

(b) (i) State and prove uniform boundedness theorem.

(ii) Prove that the normed space X of all Polynomials with norm defined by $\|x\| = \underset{i}{max} |\alpha i|$ (Where $\alpha_0, \alpha_1, \ldots$ are the co. efficients of x) is not complete.

5. (a) (i) prove that in a normal space X, strong convergence implies. Weak convergence with the same limit and the converse is not true in general. If dim X $< \infty$, prove that weak convergence implies strong convergence.

(ii) Prove that a sequence (T_n) of operators $T_n \in B(X, Y)$ where X and Y are Banach spaces is strongly operator convergent iff the sequence $(\|T_n\|)$ is bounded and the sequence $(T_n x)$ is Cauchy in Y for every x in a total subset M of X.

(*Or*)

(b) State and prove open mapping theorem.

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RW-6155 **511301**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

COMPLEX ANALYSIS

(CBCS—2004 onwards)

Time : 3 Hours Maximum : 100 Marks

$(5 \times 20 = 100)$

Answer **all** questions.

Each questions carries 20 marks.

1. (a) (i) Obtain a necessary and sufficient condition for a function $f(z) = u(z) + v(z)$ to be analytic.

(ii) State and prove Luca's theorem.

(*Or*)

(b) (i) Show that any bilinear transformation which transforms the real axis into itself can be written with real coefficients.

(ii) Prove that the cross ratio is (z_1, z_2, z_3, z_4) is real if, and only if, the four points lie on a circle or on a straight line.

2\. (a) (i) State and prove Cauchy's theorem for a rectangle.

(ii) State and derive Cauchy's integral formula.

(*Or*)

(b) (i) State and prove Taylor's theorem.

(ii) State and prove the Maximum modulus theorem.

3. (a) (i) State and prove Rouche's theorem.

(ii) Evaluate $\int_{0}^{\infty} \frac{\log x \, dx}{\left(1+x^{2}\right)}$ by the method of residues.

(*Or*)

(b) (i) If $f(z)$ is analytic in Ω, prove that $\int_{\nu} f(z)\, dz = 0$ for every cycle ν which is homologous to zero in Ω.

(ii) Prove that $pdx + qdy$ is locally exact in Ω if, and only if, $\int_{\partial R} pdx + qdy = 0$ for every rectangle $R \subset \Omega$ with sides parallel to the axes.

4. (a) (i) if u_1 and u_2 are harmonic in a region Ω, prove that $\int_{\nu} u_1 * du_2 - u_2 * du_1 = 0$ for every cycle ν which is homologous to zero in Ω.

(ii) Suppose that $u\ (z)$ is harmonic for $|z| < \mathrm{R}$, continuous for $|z| \le \mathrm{R}$. Then prove that

$$u\ (a) = \frac{1}{2\pi} \int_{|z| = R} \frac{\mathrm{R}^2 - |a|^2}{|z - a|^2}\, u\ (z)\, d\ \theta \text{ for all } |a| < \mathrm{R}.$$

(*Or*)

(b) (i) Suppose that $f_n\ (z)$ is analytic in the region Ω_n and that the sequence $\{f_n(z)\}$ converges to a limit function $f\,(z)$ in a region Ω uniformly on every compact subset of Ω. Prove that $f\,(z)$ is analytic in Ω. Further, prove that $f_n'\ (z)$ converges uniformly to $f'\,(z)$ on every compact subset of Ω.

(ii) If $f(z) = \dfrac{z+4}{(z+3)(z-1)^2}$, find Laurent's series expansions in $0 < |z-1| < 4$ and $|z-1| > 4$.

5. (a) (i) Prove that $\dfrac{\pi}{\sin \pi z} = \lim_{m \to \infty} \sum_{-m}^{+m} (-1)^n \dfrac{1}{z-n}$.

(ii) Prove that the infinite product $\prod_{1}^{\infty} (1 + a_n)$ with $1 + a_n \neq 0$ converges simultaneously with the series $\sum_{1}^{\infty} \log(1 + a_n)$ whose terms represent the values of the principal branch of the logarithm.

(*Or*)

(b) Prove that there exists an entire function with arbitrarily prescribed zeroes a_n, provided that, in the case of infinitely many zeroes, $a_n \to \infty$. Also prove that every entire function with these and no other zeroes can be written in the form

$$f(z) = z_m e^{g(z)} \prod_{n=1}^{\infty} \left(1 - \frac{z}{a_n}\right) e^{\frac{z}{a_n} + \frac{1}{2}\left(\frac{z}{a_n}\right)^2 + \dots + \frac{1}{m_n}\left(\frac{z}{a_n}\right)^{m_n}}$$

where the product is taken over all $a_n \neq o$, the m_n are certain integers and $g\ (z)$ is an entire function. Deduce that every function which is meromorphic in the whole plane is the quotient of two entire functions.

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RW-6156	511303

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

OPTIMIZATION TECHNIQUES

(CBCS—2004 onwards)

Time : 3 Hours Maximum : 100 Marks

$(5 \times 20 = 100)$

Answer **all** questions.

All questions carry equal marks.

1. (a) (i) Explain Simplex Algorithm.

(ii) Define primal - dual pair. Use two phase simplex method to

$$\text{Maximize } Z = 5x_1 + 3x_2$$

subject to the constraints :

$$2x_1 + x_2 \leq 1$$
$$x_1 + 4x_2 \geq 6$$

and $x_1, x_2 \geq 0$.

(*Or*)

(b) (i) Explain dual simplex Algorithm.

(ii) Use simplex method to solve the following L.P.P.

$$\text{Maximize } Z = 7x_1 + 5x_2$$

subject to the constraints :

$$x_1 + 2x_2 \le 6$$
$$4x_1 + 3x_2 \le 12$$

and $x_1, x_2 \ge 0$.

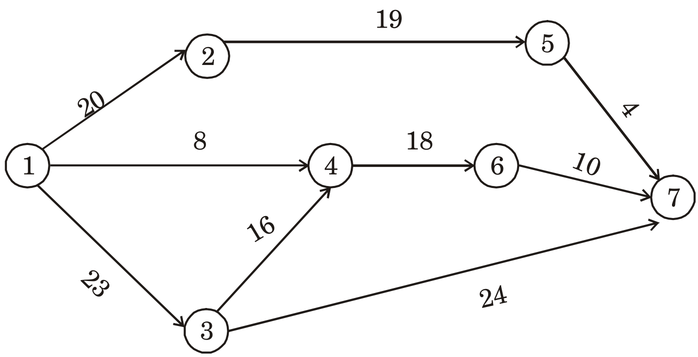
2. (a) A small maintenance project consists of the following 12 jobs.

Job (i, j)	(1, 2)	(2, 3)	(2, 4)	(3, 4)	(3, 5)	(4, 6)
Duration D_{ij}	2	7	3	3	5	3
Job	(5, 8)	(6, 7)	(6, 10)	(7, 9)	(8, 9)	(9, 10)
Duration	5	8	4	4	1	7

Determine the critical path, total float, free float and time taken for the network.

(*Or*)

(b) Find the critical path, total float, free float and time taken for the following network.

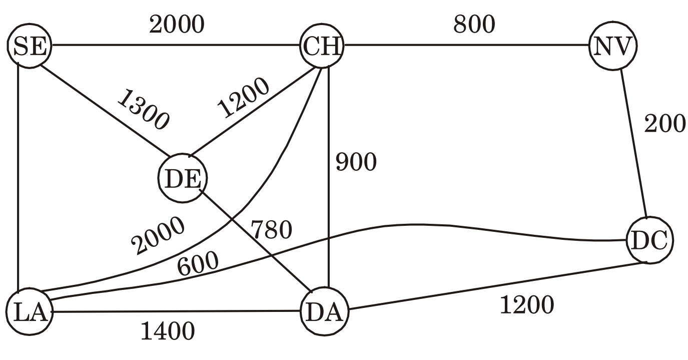


3. (a) (i) Discuss the relation between linear programming and dynamic programming. Show how to solve a linear programming problem by dynamic programming technique.

(ii) Explain minimal spanning tree problem (method).

(*Or*)

(b) Suppose it is desired to establish a cable communication network that links the major cities shown in the following network. Determine how the cities are connected such that the total used cable is minimized.



4. (a) Determine the maximum or minimum (if any) of the function $f(x) = x_1 + 2x_2 + x_1 x_2 - x_1^2 - x_2^2$.

(*Or*)

(b) (i) Use golden mean search to approximate the location of the maximum of $f(x) = x(5\pi - x)$ on [0, 20] to within $\in = 1$.

(ii) Use the Newton Raphson method to

Maximize $Z = 2x_1 + x_1 x_2 + 3x_2$

subject to $x_1^2 + x_2 = 3$

5. (a) (i) Solve numerically

Maximize

$$Z = -(2x_1 - 5)^2 - (x_2 - 3)^2 - (5x_3 - 2)^2$$

(ii) Show that the quadratic function

$$f(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}\, x_i\, x_j.$$

with symmetric coefficient matrix A, is concave if and only if A is negative semi-definite.

(*Or*)

(b) Explain non-linear algorithm in detail.

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M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

MATHEMATICAL STATISTICS

(CBCS—2004 onwards)

Time : 3 Hours Maximum : 100 Marks

(5 × 20 = 100)

Answer **all** questions.

Each questions carries 20 marks.

1. (a) (i) If $X_n \xrightarrow{r} X$, prove that

$E|X_n|^r \to E|X|^r$.

(ii) State and prove Kolmogorov's inequality.

(iii) If $\sum_{n=1}^{\infty}$ var $X_n < \infty$, prove that $\sum_{n=1}^{\infty}(X_n - EX_n)$

converges almost surely.

(*Or*)

(b) (i) Let $\{X_n\}$ be a sequence of iid rv's with common finite mean $= EX_1$. Prove that $n^{-1} S_n \xrightarrow{P}$ as $n \to \infty$.

(ii) State and prove Borel - Cantelli lemma.

2. (a) (i) If b_k is the sample central moment, find E (b_2), var (b_2), E (b_3) and E (b_4).

(ii) Let (X_1, Y_1), (X_2, Y_2), ..., (X_n, Y_n) be a sample from a bivariate population with variances σ_1^2, σ_2^2 and covariance $\rho \sigma_1 \sigma_2$. Prove that $E\left(S_1^2\right) = \sigma_1^2$, $ES_2^2 = \sigma_2^2$ and $ES_{11} = \rho \sigma_1 \sigma_2$.

(*Or*)

(b) (i) Define the Chi-square, t and f - distributions.

(ii) Let $X_1, X_2 \ldots, X_n$ be iid N (μ, σ^2) rv's. Prove that $\overline{X}$ and $\left(X_1 - \overline{X}, X_2 - \overline{X}, \ldots, X_n - \overline{X}\right)$ are independent.

(iii) Find the distribution of $(n-1)S^2/\sigma^2$.

3. (a) (i) Define a sufficient statistic and show that not every statistic is sufficient.

(ii) Obtain a necessary and sufficient condition for an unbiased estimate to be a UMVUE.

(*Or*)

(b) (i) State and prove the CRK inequality.

(ii) Explain the method of maximum likelihood estimation with an example.

4. (a) (i) State and prove the Neyman - Pearson fundamental lemma.

(ii) A die is rolled 120 times with the following results :

		1	2	3	4	5	6
Frequency	:	20	30	20	25	15	10

Test the hypothesis that the die is fair at level $\alpha = 0.05$.

(*Or*)

(b) (i) Find the likelihood ratio test of $\mu = \mu_o$ against $\neq \quad _o$ in sampling from $N(\mu, \sigma^2)$ where both μ and σ^2 are unknown.

(ii) The additional hours of sleep gained by 8 patients in an experiment with a certain drug were recorded as follows :

Patient	:	1	2	3	4	5	6	7	8
Hours gained	:	·7	–1.1	3.4	·8	2.0	·1	–·2	3.0

Assuming that these patients form a random sample from a population of such patients and that the number of additional hours gained from the drug is a normal random variable, test the hypothesis that the drug has no effect at level $\alpha = \cdot 10$.

5. (a) (i) Let $X_1, X_2, ..., X_n$ be iid rv's, $X_i \sim N(\mu, \sigma^2)$, find a $1 - \alpha$ level confidence interval for σ^2 whenever μ is unknown.

(ii) Let $X_1, X_2, \ldots, X_n$ be a sample from $N(\mu, \sigma^2)$, where σ^2 is known. Find the shortest length confidence interval for μ at level $1 - \alpha$.

(*Or*)

(b) (i) Let $X_1, X_2, \ldots, X_n$ be a sample from $U(0, \theta)$. Find a $1 - \alpha$ level confidence interval for θ.

(ii) Let $A(\theta\ 0)$ be the acceptance region of a UMP unbiased size α test of $H_0(\theta_0) : \theta = \theta_0$ against $H_1(\theta_0) : \theta \neq \theta_0$ for each θ_0. Prove that $S(x) = \{\theta \mid x \in A(\theta)\}$ is a UMA unbiased family of confidence sets at level $1 - \alpha$.

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RW-6158 **511402**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

MECHANICS

(CBCS—2004 onwards)

Time : 3 Hours Maximum : 100 Marks

(5 × 20 = 100)

Answer **all** questions.

Each questions carries 20 marks.

1. (a) (i) Explain virtual displacement. (10)

(ii) A disk of radius r and mass m can roll without slipping on a thin rod which rotates about a fixed point O at a constant rate w. Obtain an expression of the form $\mathrm{T}(q, \dot{q})$ for the total kinetic energy of the disk.

(10)

(*Or*)

(b) (i) Explain workless constraints with an example.

(10)

(ii) A thin uniform rod of mass m and length l is constrained to move in the xy plane with end A remaining on the x-axis. Using (x, θ) as generalized co-ordinates, find expressions for the kinetic energy and the generalized momentum p_θ.

(10)

2. (a) (i) Find the differential equations of motion for a spherical pendulum of length l.

(10)

(ii) Consider a natural system with $T = \frac{1}{2} m(q) \dot{q}^2$ and $v = v(q)$. Show that the correct differential equation of motion is obtained by equating to zero the time derivative of the total energy $E = T + V$.

(10)

(*Or*)

(b) (i) Explain Liouville's system. (10)

(ii) A particle of mass m is connected by a massless spring of stiffness K and unstressed length r_o to a point P which is moving along a circular path of radius 'a' at a uniform angular rate w. Assuming that the particle moves without friction on a horizontal plane, find the differential equation of motion.

(10)

3. (a) (i) Derive Euler- Lagrange equation. (10)

(ii) Derive Hamilton's canonical equations by starting with the modified Hamilton's principle and applying the Euler - Lagrange equations in a phase space of $2n$ dimensions.

(10)

(*Or*)

(b) (i) Given a holonomic system with a Lagrangian function L = $\frac{1}{2} m \left(\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2 \right) - mgx_3$ and a constraint

$$\left(\dot{x}_1 - \dot{x}_2 + \dot{x}_3 = 0 \right)$$

Use an augmented Lagrangian function to obtain the differential equations of motion. Solve for $\ddot{x}_1$.

(10)

(ii) Given a mass - spring system consisting of a mass m and a linear spring of stiffness K. Find the equations of motion using the Hamiltonian procedure. Assume that the displacement x is measured from the unstressed position of the spring.

(10)

4. (a) (i) State and prove Stackel's theorem. (10)

(ii) Consider a Standard Hamiltonian system with $H = q + p$. Assuming separability, solve the Hamiltonian - Jacobi equation and show that $s = -\alpha t + \alpha q - \frac{q^2}{2}$ is a solution. Show that this principal function leads to the correct solution of the canonical equations.

(10)

(*Or*)

(b) (i) Derive modified Hamilton - Jacobi equation.

(10)

(ii) Explain briefly Pfaffian differential forms.

(10)

5. (a) (i) A particle of mass m moves in the xy plane under the action of a potential function $v = ky$. For a homogeneous point transformation

$$Q_1 = xy.\ Q_2 = \frac{1}{2}\left(x^2 - y^2\right)$$

find the expression for p_1 and p_2 and the generating function $F_2(q, p)$. What is the new Hamiltonian function K (Q, P) ?

(10)

(ii) Consider the transformation.

$$Q = \sqrt{e^{-2q} - p^2},\ p = \cos^{-1}(pe^q)$$

Use the poission bracket to show that it is cononical.

(10)

(*Or*)

(b) (i) Consider the transformation

$$Q = \frac{1}{2}\left(q^2 + p^2\right),\ p = -\tan^{-1}\frac{p}{q}.$$

Show that this transformation is canonical.

(10)

(ii) Given a system with $H = \frac{1}{2}\left(q^2 + p^2\right)$. Employ the Hamilton Jacobi method to solve for the motion, using first F_2 (q, p, t) and then F_4 (p, p, t). Note that p is identical with α and Q with β in each case.

(10)

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RW-6159 **511504**

M.Sc. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS—2004 onwards)

Time : 3 Hours Maximum : 100 Marks

$(5 \times 20 = 100)$

Answer **all** questions.

Each questions carries 20 marks.

1. (a) (i) Calculate the curvature and torsion of the cubic curve given by $r\ (u, u^2, u^3)$.

(10)

(ii) Find the equation of the osculating plane at a general point on the cubic curve given by $r = (u, u^2, u^3)$ and show that the osculating planes at any three points of the curve meet at a point lying in the plane determined by these three points.

(10)

(*Or*)

(b) (i) Show that the length of the common perpendicular 'd' of the tangents at two near points distance S apart is approximately given by $d = \frac{K\tau S^3}{12}$.

(10)

(ii) Find the curvature and torsion of the curves given by $r = \left\{a\left(3u - u^3\right), 3au^2, a\left(3u + u^3\right)\right\}$.

2. (a) Define

(i) Osculating circle.

(ii) Locus of the centre of spherical curvature.

(iii) Intrinsic equations.

(iv) Helices.

(4 × 5 = 20)

(*Or*)

(b) (i) Show that an involute of C is obtained by unwinding a string initially stretched along the curve, so that the string always remains taut.

(10)

(ii) State and prove fundamental existence theorem for space curves.

(10)

3. (a) (i) Describe about the surfaces of revolution.

(10)

(ii) A helicoid is generated by the screw motion of a straight line skew to the axis. Find the curve coplanar with the axis which generates the same helicoid.

(10)

(*Or*)

(b) (i) Prove that if θ is the angle at the point (u, v) between the two directions given by $P\,du^2 + 2Q\,du\,dv + R\,dv^2 = 0$ then

$$\tan\theta = \frac{2H\left(Q^2 - PR\right)^{1/2}}{ER - 2FQ + GP}.$$

(10)

(ii) Explain about the intrinsic properties.

(10)

4. (a) (i) Prove that every helix on a cylinder is a geodesic.

(10)

(ii) Prove that a geodesic disk of greater radius is convex but not simple.

(10)

(*Or*)

(b) (i) Prove that if (λ, μ) is the geodesic curvature vector, then

$$K_g = \frac{H\lambda}{Fu' + Gv'} = \frac{H\mu}{Eu' + Fv'}.$$

(10)

(ii) Prove that the orthogonal trajectories of the curves v = constant are geodesics then H^2/E is independent of u.

(10)

5. (a) (i) State and prove Meusnier's theorem. (10)

(ii) Derive the Dupin Indicatrix. (10)

(*Or*)

(b) (i) Prove that a necessary and sufficient condition for a surface to be developable is that its Gaussian curvature will be zero.

(10)

(ii) Prove that a necessary and sufficient condition that a curve on a surface be a line of curvature is that the surface normals along the curve form a developable.

(10)

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RW-6160 **541001**

M.A. DEGREE EXAMINATION, NOVEMBER 2010

Mathematics

OFFICE AUTOMATION (COMPULSORY)

(CBCS—2004 onwards)

Time : 3 Hours Maximum : 100 Marks

($5 \times 20 = 100$)

Answer all **five** questions.

All questions carry **equal** marks.

1. (a) Explain Various MS-OFFICE programs.

(b) Explain various editing options in MS-Office programs.

(*Or*)

(c) Explain the Tool Bars available in the opening window.

(b) Write down the steps to add a program to the Shortcut bar.

2. (a) Explain the Auto Formatting option in MS-Word.

(b) Explain the Mail Merge feature of MS-Word with an example.

(*Or*)

(c) Write down the steps to insert page break in a document.

(d) Explain how will you add Headers and Footers to a document.

3. (a) Explain how you will enter, edit and copy formula in a Worksheet.

(b) Explain how will you change the background of a Worksheet.

(*Or*)

. (c) Explain any TEN functions in Excel with example.

(d) Write down the steps to create a Chart.

4. (a) How will you create a table using Table Wizard ?

(b) Write the steps to create and edit a Query.

(*Or*)

. (c) In MS-ACCESS how will you add, insert and delete records.

(d) Write down the steps to create a Report in MS-ACCESS.

5. (a) Write the steps to create a presentation using auto content wizard.

(b) How will you prepare Handouts ?

(*Or*)

. (c) How will you Create a Binder ?

(d) Explain features of MS-Outlook.

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