Printed Pages 4 Roll No

300114(14)

B. E. PQ (First & Second Semester) Examination
Nov.-Dec. 2019

(Old Scheme)

(AEI, Bio Tech, Chem., Civil, CSE, Elect., EEE, EI, Et & T, IT, Mech., Mining, Metallurgy, Mechatronics, Production, Automobile & Lateral Diploma Branch)

APPLIED MATHEMATICS - I

Time Allowed : Three hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : *Attempt all questions. Part (a) of each question is compulsory and carries 2 marks. Attempt any two parts from (b), (c) and (d) of each question which carry 7 marks each.*

Unit - I

1. (a) Define rank of a matrix.

(b) Find the rank of the matrix :

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$$

(c) Test for consistency and solve if found consistent :

$2x + y + 5z = 4$

$3x - 2y + 2z = 2$

$5x - 8y - 4z = 1$

(d) Find the characteristic equation of the matrix :

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

and hence find A^{-1}.

Unit - II

2. (a) State Leibnitz's theorem on successive differentiation.

(b) If $y = \dfrac{1}{x^2 + a^2}$,

find y_n.

(c) Expand $\sin x$ in powers of $x - \frac{\pi}{2}$. Hence find the value of $\sin 91^\circ$ correct to 4 decimal places.

(d) Trace the curve :

$y^2(2a - x) = x^3$

Unit - III

3. (a) Evaluate $\int_0^{\pi/2} \sin^9 x \, dx$

(b) Prove that :

$$\int_0^{\pi/2} \cos^n x \cos nx \, dx = \frac{\pi}{2^{n+1}}$$

(c) Find the area of the circle $x^2 + y^2 = a^2$.

(d) Find the perimeter of the cardioid $r = a(1 + \cos\theta)$.

Unit - IV

4. (a) If $u = f\left(\frac{y}{x}\right)$, show that

$x u_x + y u_y = 0$.

(b) Verify Euler's theorem for the function :

$u = x^3 \log\left(\frac{y}{x}\right)$.

(c) Discuss the maximum or minimum values of the function :

$f(x, y) = x^3 - 4xy + 2y^2$

(d) Evaluate

$$\int_0^{\infty} \frac{\tan^{-1} ax}{x(1 + x^2)}$$

Unit - V

5. (a) Write the condition for exactness of the differential equation

$M \, dx + N \, dy = 0$

where M and N are both functions of x and y.

(b) Solve $p^3 - 4xy\,p + 8y^2 = 0$.

(c) Solve

$$y^2 + x^2\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} = 4\left(\frac{dx}{dy}\right)^2$$

(d) The rate of cooling of a body is proportional to the difference between temperature of the body and surrounding air. If the air temperature is 20°C and the body cools for 20 minutes from 140°C to 80°C, find when the temperature will be 35°C.