MATHEMATICS

Time allowed : 3 hours ***Maximum Marks : 100***

General Instructions :

(i) *The question paper consists of three sections A, B and C.* ***Section A is compulsory*** *for all students. In addition to Section A, every student has to attempt* ***either Section B OR Section C.***

(ii) ***For Section A***

Question numbers 1 to 8 are of 3 marks each.

Question numbers 9 to 15 are of 4 marks each.

Question numbers 16 to 18 are of 6 marks each.

(iii) ***For Section B/Section C***

Question numbers 19 to 22 are of 3 marks each.

Question numbers 23 to 25 are of 4 marks each.

Question number 26 is of 6 marks.

(iv) *All questions are compulsory.*

(v) *Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.*

(vi) *Use of calculator is not permitted. However, you may ask for logarithmic and statistical tables, if required.*

QUESTION PAPER CODE 65/1/1

SECTION 'A'

1. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, prove that $A^3 - 4A^2 + A = 0$.

2. Show that

$$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0,$$

where a, b, c are in A.P.

3. In a single throw of three dice, determine the probability of getting

(a) a total of 5, (b) a total of at most 5.

4. A class consists of 10 boys and 8 girls. Three students are selected at random. Find the probability that the selected group has

(i) all boys,

(ii) all girls,

(iii) 2 boys and 1 girl.

5. Evaluate :

$$\int \frac{\sin 2x}{a^2 \sin^2 x + b^2 \cos^2 x} dx$$

6. Evaluate :

$$\int \frac{\sqrt{16 + (\log x)^2}}{x} dx$$

7. Form the differential equation representing the family of curves $y^2 - 2ay + x^2 = a^2$, where a is an arbitrary constant.

8. Solve the following differential equation :

$$(1 + x^2)\frac{dy}{dx} - 2xy = (x^2 + 2)(x^2 + 1)$$

$S_1 : p \vee q$

OR

Solve the following differential equation :

$$x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$$

9. Prove that :

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$

OR

Test the validity of the following argument :

$S_2 : \sim p$

$S : q$

10. Evaluate :

$$\lim_{y \to 0} \frac{(x + y)\sec(x + y) - x \sec x}{y}$$

OR

Evaluate :

$$\lim_{x \to 0} \frac{x\,[1-\sqrt{1-x^2}\,]}{\sqrt{1-x^2}\,(\sin^{-1} x)^3}$$

11. Differentiate $\sin\sqrt{x}$ w.r.t. x from first principles.

12. If $x = a\left(\dfrac{1+t^2}{1-t^2}\right)$ and $y = \dfrac{2t}{1-t^2}$, find $\dfrac{dy}{dx}$.

13. The surface area of a spherical bubble is increasing at the rate of 2 cm^2/sec. Find the rate at which the volume of the bubble is increasing at the instant its radius is 6 cm.

14. Evaluate :

$$\int \frac{2x-1}{(x-1)(x+2)(x-3)}\,dx$$

15. Evaluate :

$$\int_0^{\pi} \frac{dx}{5+4\cos x}$$

16. Using matrices, solve the following system of linear equations :

$$x + y + z = 4$$

$$2x + y + 3z = 19$$

OR

If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y such that $A^2 + x\,I = yA$.

Hence find A^{-1}.

17. A wire of length 36 cm is cut into two pieces. One of the pieces is turned in the form of a square and the other in the form of an equilateral triangle. Find the length of each piece so that the sum of the areas of the two be minimum.

18. Find the area bounded by the curve $x^2 = 4y$ and the straight line $4y - 2 = x$

OR

Evaluate the following as limit of sums :

$$\int_0^2 (x^2 + x)\,dx$$

Section B

19. Express the vactor $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$ as sum of two vactors such that one is parallel to the vector $\vec{b} = 3\hat{i} + \hat{k}$ and the other is perpendicular to $\vec{b}$.

20. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ be coplanar, show that $c^2 = ab$.

21. A car, travelling with a uniform acceleration, has a velocity of 18 km/hour at a certain time and 54 km/hour after covering a distance of 500 m. How much further will it travel to attain a velocity of 72 km/hour ?

22. A body falls freely from the top of a tower. It covers $\frac{5}{9}$th of the whole distance in the last second. Find the height of the tower and the total time taken by the body to fall down.

OR

A cricket ball is projected with a velocity of 29·4 m/sec. Find

(i) the greatest range on the horizontal plane; and

(ii) the angle of projection to give a range of 44.10 m.

$C\,(2,\ -3,\ -1)$

23. Find the co-ordinates of the foot of the perpendicular drawn from the point A (1, 8, 4) to the line joining the points $B\,(0, -1, 3)$ and .

24. The resultant of two forces and acting at a point is at right angles to force $\vec{P}$, while the resultant of forces and , acting at the same angle, is at right angles to force $\vec{S}$. Prove that $P = \sqrt{SQ}$.

25. $\vec{P}$ and are two unlike parallel forces. When the magnitude of $\vec{P}$ is doubled, it is found that the line of action of is midway between the lines of action of the new and the original resultants. Find the ratio of P and Q.

OR

Three forces, acting on a particle, are in equilibrium. If the angle between the first force and the second force be 120° and that between the second force and the third force be 135°, find the ratio of their magnitudes.

26. Find the cartesian as well as the vector equation of the planes passing through the intersection of the planes $\vec{r}.(2\hat{i} + 6\hat{j}) + 12 = 0$ and $\vec{r}.(3\hat{i} - \hat{j} + 4\hat{k}) = 0$ which are at unit distance from the origin.

SECTION C

19. Solve the following linear programming problem graphically :

Maximise z = 60x + 15y

subject to constraints

$x + y \leq 50$

20. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants per day while B can stitch 10 shirts and 4 pants per day. Form a linear programming problem to minimise the labour cost to produce at least 60 shirts and 32 pants.

21. A company has two plants to manufacture motor cycles. 70% motor cycles are manufactured at the first plant, while 30% are manufactured at the second plant. At the first plant, 80% motor cycles are rated of the standard quality while at the second plant, 90% are rated of standard quality. A motor cycle, randomly picked up, is found to be of standard quality. Find the probability that it has come out from the second plant.

22. The probability that a student entering a university will graduate is 0.4. Find the probability that out of 3 students of the university :

[Use $e^{-1} = 0.3679$]

(i) none will graduate,

(ii) only one will graduate,

(iii) all will graduate

OR

In a book of 200 pages, 200 misprints are randomly distributed. Using Poisson's distribution calculate the probability that a randomly observed page of the book will be found to have at least 3 errors.

23. A, B and C are engaged in a printing business. A being the working partner, receives 10% of the net profit as salary. The remaining profit is divided among themselves in the ratio 4 : 5 : 9. If A gets in total Rs. 3,00,000, find the total profit in the business and the shares of B and C in it.

OR

A and B are partners in a business sharing profits and losses equally. They admit a new partner C and it is agreed that now the profits and losses will be shared amongst A, B and C in the ratio 9 : 8 : 7 respectively. If C paid Rs. 2.10 lakh as premium for the goodwill, find the shares of A and B in the premium.

24. Find the present worth of an ordinary annuity of Rs. 1200 per annum for 10 years at 12% per annum, compounded annually. $[\text{Use}: (1.12)^{-10} = 0.3221]$

25. A calculator manufacturing company finds that the daily cost of producing x calculators is given by C(x) = 200x + 7500.

    (i) If each calculator is sold for Rs. 350, find the minimum number of calculators that must be produced daily and sold to ensure no loss.

    (ii) If the selling price is increased by Rs. 150, what would be the break-even point ?

26. A bill was drawn on April 4, 2004 at 8 months after date and was discounted on July 14, 2004 at 5% per annum. If the Banker's gain is Rs. 200, find the face value of the bill.

QUESTION PAPER CODE 65/1

SECTION 'A'

1. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $f(x) = x^2 - 2x - 3$, show that $f(A) = 0$.

2. Using properties of determinants, solve for x :

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

3. An integer is chosen at random from the first 200 positive integers. Find the probability that it is divisible by 6 or 8.

4. X is taking up subjects — Mathematics, Physics and Chemistry in the examination. His probabilities of getting Grade A in these subjects are 0.2, 0.3 and 0.5 respectively. Find the probability that he gets.

    (i) Grade A in all subjects;

    (ii) Grade A in no subject;

    (iii) Grade A in two subjects.

5. Evaluate :

$$\int \frac{\sin 2x}{(a + b\cos x)^2}\, dx$$

6. Evaluate :

$$\int \left[\frac{1}{\log x} - \frac{1}{(\log x)^2} \right] dx$$

7. Solve the following differential equation :

$$\frac{dy}{dx} + y \cot x = x^2 \cot x + 2x$$

8. Solve the following differential equation :

$$(x^2 + xy)\, dy = (x^2 + y^2)\, dx$$

OR

Solve the following differential equation :

$$\frac{d^2y}{dx^2} = e^x + \cos x, \text{ given that } \frac{dy}{dx} = 1 = y, \text{ when } x = 0.$$

9. Test the validity of the following argument :

$$S_1 : p \vee q; \quad S_2 :\sim p; \quad S :\sim q$$

OR

If B is a Boolean Algebra and $x, y \in B$, then show the following :

$$(x + y) + (x' \,.\, y') = 1$$

10. Evaluate :

$$\lim_{y \to 0} \frac{(x+y)\sec(x+y) - x \sec x}{y}$$

11. Differentiate $\tan\sqrt{x}$ w.r.t. x from first principles.

12. If $y = \left\{ x + \sqrt{x^2 + a^2} \right\}^n$, prove that

$$\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}$$

13. Find the intervals in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly increasing or decreasing. Also find the points on which the tangents are parallel to the x-axis.

14. Evaluate :

$$\int \frac{x^2}{x^2 + 6x + 12}\, dx$$

15. Evaluate :

$$\int_{-5}^{0} f(x)\,dx \text{, where } f(x) = |x| + |x+2| + |x+5|.$$

16. Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius 12 cm is 16 cm.

OR

Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if

17. Using matrix method solve the following system of linear equations :

$$x + y - z = 1$$

18. Find the area of the region bounded by the curve and the line y = x.

OR

Find the area enclosed by the parabola $y^2 = x$ and line y + x =2.

SECTION B

$3x - y + 2z = 13$

19. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of equal magnitude, find the angle between $\vec{a}$ and $(\vec{a} + \vec{b} + \vec{c})$.

20. Show that the four points A, B, C and D, whose position vectors are $6\hat{i} - 7\hat{j}$, $16\hat{i} - 19\hat{j} - 4\hat{k}$, $3\hat{j} - 6\hat{k}$ and $2\hat{i} - 5\hat{j} + 10\hat{k}$ respectively, are coplanar.

21. A body moves for 3 seconds with a uniform acceleration and describes a distance of 108 m. At that point the acceleration ceases and the body covers a distance of 126 m in the next 3 seconds. Find the initial velocity and acceleration of the body.

22. A body is projected with a velocity of 24 m/sec at an angle of 60° with the horizontal. Find

(i) the equation of its path;

(ii) its time of flight; and

(iii) the maximum height attained by it.

OR

A particle is projected so as to graze the top of two walls, each of height 10 m, at 15 m and 45 m, respectively from the point of projection. Find the angle of projection.

23. Find the equation of the line passing through the point $P(-1, 3, -2)$ and prependicular to lines and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.

24. The resultant of forces and acting at a particle is $\vec{R}$. If is doubled, $\vec{R}$ is doubled. If is reversed, $\vec{R}$ is again doubled. Prove that

OR

A and B are two fixed points in a horizontal line at a distance 50 cm apart. Two fine strings AC and BC of length 30 cm and 40 cm respectively support a weight W at C. Show that the tensions in the strings CA and CB are in the ratio of 4: 3.

25. The resultant of two unlike parallel forces of 18 N and 10N acts along a line at a distance of 12 cm from the line of action of the smallar force. Find the distance between the lines of action of the two given forces.

26. Find the equation of the sphere passing through the points $(1, -3, 4), (1, -5, 2),$ and having its centre on the plane

SECTION C

[illegible]

19. A speaks the truth 8 times out of 10 times. A die is tossed. He reports that it was 5. What is the probability that it was actually 5 ?

20. A coin is tossed 4 times. Find the mean and variance of the probability distribution of the number of heads.

OR

For a Poisson distribution, it is given that P (X = 1) = P (X = 2). Find the value of the mean of the distribution. Hence find P (X = 0) and P (X = 4).

21. If the banker's gain on a bill be $\frac{1}{9}$th of banker's discount, the rate of interest being 10% per annum, find the unexpired period of the bill.

22. A bill of Rs. 5300, drawn on 16th January, 2003 for 8 months was discounted on 12th February, 2003 at 10% per annum. Find the banker's gain and discounted value of the bill.

23. In a business partnership, A invests half of the capital for half of the period, B invests one-third of the capital for one-third of the period, and C invests the rest of the capital for the whole period. Find the share of each in the total profit of Rs. 1,90,000.

24. A plans to buy a new flat after 5 years, which will cost him Rs. 5,52,000. How much money should he deposit annually to accumulate this amount, if he gets interest 5% per annum compounded annually ? [Use : $(1.05)^5 = 1.276$]

25. The cost function of a firm is given by $C(x) = 300x - 10x^2 + \frac{1}{3}x^3$, where x stands for the output.

Calculate :

(i) the output at which the marginal cost is minimum;

(ii) the output at which the average cost is equal to the marginal cost.

OR

The total cost and the total revenue of a firm that produces and sells x units of its product daily are expressed as

$C(x) = 5x + 350$ and

$R(x) = 50x - x^2$.

Calculate :

(i) the break-even points, and

(ii) the number of units the firm will produce which will result in loss.

26. A manufacturer produces two types of steel trunks. He has two machines A and B. The first type of the trunk requires 3 hours on machine A and 3 hours on machine B. The second type of trunk requires 3 hours on machine A and 2 hours on machine B. Machines A and B are run daily for 18 hours and 15 hours respectively. There is a profit of Rs. 30 on the first type of the trunk and Rs. 25 on the second type of the trunk. How many trunks of each type should be produced and sold to make maximum profit ?

Marking Scheme — Mathematics

General Instructions :

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. Marks may not be deducted in questions on integration if constant of integration is not written.

5. In question(s) on differential equations, constant of integration has to be written.

6. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

7. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

EXPECTED ANSWERS/VALUE POINTS

SECTION 'A'

1. $\text{LHS} = A\left[A^2 - 4A + I\right]$ ½ m

Now $A^2 - 4A + I$ 1 m

$$= \begin{bmatrix} 7-8+1 & 12-12 \\ 4-4 & 7-8+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$ 1 m

$$\therefore \quad A\left[A^2 - 4A + I\right] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \text{RHS}$$ ½ m

2. $$A = \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$$\therefore A = \begin{vmatrix} & & \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$$

$R_1 \to R_1 + R_3 - 2R_2$

$$A = \begin{vmatrix} 0 & 0 & a+c-2b \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$ 2 m

If a, b and c are in A.P. then $2b = a + c$ ½ m

½ m

3. (a) Let A be the event of getting a total of $5 \to (113,\ 131,\ 122,\ 221,\ 212,\ 311)$ 1 m

$$\therefore P(A) = \frac{6}{216} = \frac{1}{36}$$

½ m

(b) Let B be the event of getting at most sum 5

This can be obtained as

$[(1,1,1), (1,2,1), (1,1,2), (2,1,1), (1,1,3), (1,3,1), (1,2,2), (2,2,1), (2,1,2), (3,1,1)]$ 1 m

$\therefore \quad P(B) = \frac{10}{216} = \frac{5}{108}$ ½ m

4. Total number of student = 10 + 8 = 18

$\therefore$ (i) P (All boys) (½+½) m

(ii) P (All girls) $= \frac{8C_3}{18C_3} = \frac{7}{102}$ (½+½) m

(iii) P (2boys, 1girl) $= \frac{10C_2 \times 8C_1}{16C_3} = \frac{15}{34}$ (½+½) m

5. Let $a^2 \sin^2 x + b^2 \cos^2 x = t$ $= \frac{10C_3}{18C_3} = \frac{5}{34}$

$\therefore \quad 2a^2 \sin x \cos x - 2b^2 \sin x \cos x = \frac{dt}{dx}$ 1 m

or $(a^2 - b^2) . \sin 2x \; dx = dt$

$\therefore \quad I = \int \frac{dt}{(a^2 - b^2)t} = \frac{1}{a^2 - b^2} \log |t| + C$ 1 m

$= \frac{1}{(a^2 - b^2)} \log \left| a^2 \sin^2 x + b^2 \cos^2 x \right| + C$ 1 m

6. Let $\log x = t \quad \Rightarrow \frac{dx}{x} = dt$ ½ m

$\therefore \quad I = \int \sqrt{16 + t^2} dt$ ½ m

$$= \frac{1}{2} t \sqrt{16 + t^2} + 8 \log \left| t + \sqrt{16 + t^2} \right| + C$$ 1 m

$$= \frac{1}{2} \log x \sqrt{16 + (\log x)^2} + 8 \log \left| \log x + \sqrt{16 + (\log x)^2} \right| + C$$ 1 m

7. $y^2 - 2ay + x^2 = a^2$ ....(i)

$\therefore \quad 2y \frac{dy}{dx} - 2a \frac{dy}{dx} + 2x = 0$ ½ m

or $\quad a = \dfrac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}$ ....(ii) 1 m

Putting this value of a in (ii), we get

$$y^2 - 2y \frac{\left(x + y\frac{dy}{dx}\right)}{\frac{dy}{dx}} + x^2 = \left(\frac{x + \frac{dy}{dx}.y}{\frac{dy}{dx}} \right)^2$$

$$y.\frac{1}{1+x^2} = \int \frac{x^2+2}{x^2+1}\, dx = \int dx + \int \frac{dx}{1+x^2}$$ ½ m

or $\quad (x^2 - 2y^2)\left(\frac{dy}{dx}\right)^2 - 4xy \frac{dy}{dx} - x^2 = 0$ 1 m

8. The given differential equation can be written as

$$\frac{dy}{dx} - \frac{2x}{(1+x^2)} . y = x^2 + 2$$

$\therefore \quad$ I.F. $= \dfrac{1}{1+x^2}$ 1 m

$\therefore \quad$ The solution is

1 m

$= x + \tan^{-1} x + C$ 1 m

OR

$$x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$$

This is a homogeneous differential equation

Putting $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ ½ m

$\therefore \quad x\left[v + x\dfrac{dv}{dx}\right] - vx = x\sqrt{1+v^2}$ 1 m

$$\text{or} \quad \int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$$

$$\Rightarrow \quad \log|cx| = \log\left|v + \sqrt{1+v^2}\right|$$

or $\quad v + \sqrt{1+v^2} = cx$ 1 m

or $\quad y + \sqrt{x^2+y^2} = cx^2$ ½ m

9. The statement results in following Truth Table

p	q	$p \to q$			$(p \to q) \wedge (q \to p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Truth Table 3 m

1 m

OR

	S	S_2	S_1	S
p	q	~ p		q
T	T	F	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

Truth Table 2½ m

Identifying Critical Row 1 m

The given argument is valid ½ m

10. ½ m

$$= \lim_{y \to 0} \frac{x\left[\cos x - \cos (x+y)\right]}{y \cos x \cos (x+y)} + \sec x$$ 1 m

$$= \lim_{y \to 0} \frac{x.2\sin\left(\frac{2x+y}{2}\right).\sin\frac{y}{2}}{2\cos x .\cos (x+y).\frac{y}{2}} + \sec x$$ 1 m

$$= x \tan x \sec x + \sec x$$

1½ m

OR

½ m

$\therefore$ The given problem becomes

Let $x = \tan\theta$

$$\lim_{y \to 0} \frac{(x+y)\sec(x+y) - x\sec x}{y} = \lim_{y \to 0}\left[\frac{x\{\sec(x+y) - \sec x\}}{y} + \sec\right.$$

or $$\lim_{\theta \to 0} \frac{\sin\theta}{\cos\theta}.\frac{(1-\cos\theta)}{\theta^3}$$ 1½ m

$$= \lim_{\theta \to 0} \frac{\tan\theta}{\theta}.\lim_{\theta \to 0} \frac{2.\sin^2\frac{\theta}{2}}{4.\left(\frac{\theta}{2}\right)^2}$$ 1½ m

$$= 1.\frac{1}{2} = \frac{1}{2}$$ ½ m

11. $\Delta y = \sin\sqrt{x + \Delta x} - \sin\sqrt{x}$

$$\therefore \quad \frac{\Delta y}{\Delta x} = \frac{\sin\sqrt{x+\Delta x} - \sin\sqrt{x}}{\Delta x}$$ ½ m

$$\therefore \quad \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{2\cos\left(\frac{\sqrt{x+\Delta x}+\sqrt{x}}{2}\right).\sin\left(\frac{\sqrt{x+\Delta x}-\sqrt{x}}{2}\right)}{\Delta x}$$ ½ m

$$= \lim_{\Delta x \to 0} \frac{\sin\left(\frac{\sqrt{x+\Delta x}-\sqrt{x}}{2}\right)}{\left(\frac{\sqrt{x+\Delta x}-\sqrt{x}}{2}\right)} \times \frac{\sqrt{x+\Delta x}-\sqrt{x}}{2} \times \frac{2\cos\left(\frac{\sqrt{x+\Delta x}+\sqrt{x}}{2}\right)}{\Delta x}$$ 1 m

$$= \lim_{\Delta x \to 0} \frac{\frac{\sin\sqrt{x+\Delta x}-\sqrt{x}}{2}}{\frac{\sqrt{x+\Delta x}-\sqrt{x}}{2}} \times \frac{\Delta x}{\sqrt{x+\Delta x}+\sqrt{x}} \times \frac{\cos\left(\frac{\sqrt{x+\Delta x}+\sqrt{x}}{2}\right)}{\Delta x}$$ 1 m

$$= 1.\frac{1}{2\sqrt{x}}\cos\sqrt{x} = \frac{1}{2\sqrt{x}}.\cos\sqrt{x}$$ 1 m

12. $x = \dfrac{a\left(1+t^2\right)}{1-t^2}$ $\qquad y = \dfrac{2t}{\left(1-t^2\right)}$

$$\therefore \frac{dx}{dt} = a.\frac{\left(1-t^2\right).2t+2t(1+t^2)}{\left(1-t^2\right)^2} = \frac{4at}{\left(1-t^2\right)^2}$$ 1½ m

$$\frac{dy}{dt} = 2.\frac{\left(1-t^2\right).1+t.2t}{\left(1-t^2\right)^2} = \frac{2+2t^2}{\left(1-t^2\right)^2} = \frac{2\left(1+t^2\right)}{\left(1-t^2\right)^2}$$ 1½ m

$$\frac{dy}{dx} = \frac{2\left(1+t^2\right)}{\left(1-t^2\right)^2}.\frac{\left(1-t^2\right)^2}{4at} = \frac{1+t^2}{2at}$$ 1 m

13. Let s be the surface area of the spherical bubble

$$\therefore \frac{ds}{dt} = 2cm^2/sec.$$ ½ m

Now $s = 4\pi r^2$

$$\Rightarrow \frac{ds}{dt} = 4\pi.\,2r.\frac{dr}{dt}$$

$$\text{or} \quad 2 = 8\pi r\frac{dr}{dt} \quad \Rightarrow \quad \frac{dr}{dt} = \frac{1}{4\pi r}$$ 1 m

$$v = \text{Volume of bubble} = \frac{4}{3}\pi r^3$$

$$\therefore \quad \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$= 4\pi r^2 \frac{1}{4\pi r} = r$$ 1½ m

$$\left(\frac{dv}{dt}\right)_{\text{at } r=6} = 6\ \text{cm}^3/\text{sec}$$ 1 m

14. $$I = \int \frac{2x-1}{(x-1)(x+2)(x-3)}\, dx$$

Let $\dfrac{2x-1}{(x-1)(x+2)(x-3)} = \dfrac{A}{x-1} + \dfrac{B}{x+2} + \dfrac{C}{x-3}$ ½ m

Getting $A = \dfrac{-1}{6}$, $B = \dfrac{-1}{3}$ and $C = \dfrac{1}{2}$ 1½ m

$$\therefore \quad I = \frac{-1}{6}\int \frac{dx}{x-1} - \frac{1}{3}\int \frac{dx}{x+2} + \frac{1}{2}\int \frac{dx}{x-3}$$ ½ m

$$= \frac{-1}{6} \log|x-1| - \frac{1}{3} \log|x+2| + \frac{1}{2} \log|x-3| + C$$ 1½ m

15. $$I = \int_0^{\pi} \frac{dx}{5+4\cos x} = \int_0^{\pi} \frac{dx}{5+4\left(\dfrac{1-\tan^2 \frac{x}{2}}{1+\tan^2 \frac{x}{2}}\right)}$$ 1m

$$I = \int_0^{\pi} \frac{\sec^2 \frac{x}{2}}{9+\tan^2 \frac{x}{2}}\, dx$$ 1 m

Let $\tan \dfrac{x}{2} = t \quad \Rightarrow \quad \dfrac{1}{2}\sec^2 \dfrac{x}{2}\, dx = dt$ ½ m

$$\therefore \quad I = \int_0^{\infty} \frac{2dt}{3^2 + t^2} = \frac{2}{3}\left[\tan^{-1} \frac{t}{3}\right]_0^{\infty}$$ 1 m

$$= \frac{2}{3} \cdot \frac{\pi}{2} = \frac{\pi}{3}$$ ½ m

16. Writing the given equation as

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ -9 \end{pmatrix} \quad \text{or} \quad AX = B \;\Rightarrow\; X = A^{-1}B$$ 1 m

$|A| = 14$ 1 m

(For every four correct co-factors,
one mark may be given) 2 m

$$\therefore \quad A^{-1} = \frac{1}{14}\begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}$$ ½ m

$$\therefore \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{14}\begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix} \begin{pmatrix} 4 \\ -1 \\ -9 \end{pmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \quad x = 8$$

$$A^2 = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$$

$$\text{Adj. } A = \begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}$$

$\therefore \quad x = -1, \;\; y = 2, \;\; z = 3$ 1½ m

OR

1 m

$\therefore \quad A^2 + xI = yA$

$$\Rightarrow \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 16+x & 8 \\ 56 & 32+x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$ 2 m

$\therefore \quad y = 8 \quad \text{and} \quad 16 + x = 3y = 24$

1 m

or $A + 8A^{-1} = 8I$ or $A^{-1} = \frac{1}{8}[8I - A]$ 1 m

$$\therefore \quad A^{-1} = \frac{1}{8}\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix} \text{ or } \begin{bmatrix} \frac{5}{8} & \frac{-1}{8} \\ \frac{-7}{8} & \frac{3}{8} \end{bmatrix}$$ 1 m

17. Let the length of two pieces be x and 36 – x (in cms) ½ m

Let the piece of length x be turned into a square and the other into an equilateral triangle.

$\therefore$ side of square $= \frac{x}{4}$, side of an equilateral triangle $= \frac{36-x}{3}$ 1 m

Area of square $= \frac{x^2}{16}$ and Area of equilateral triangle $= \frac{\sqrt{3}}{4}\left(\frac{36-x}{3}\right)^2$ 1 m

$\therefore$ Let A be the sum of the areas of two

½ m

$\therefore \quad A = \frac{x^2}{16} + \frac{\sqrt{3}}{36}(36-x)^2$

$\therefore \quad \frac{dA}{dx} = \frac{x}{8} + \frac{\sqrt{3}}{18}(36-x)(-1)$ 1 m

$\therefore \quad \frac{dA}{dx} = 0 \Rightarrow \frac{x}{8} = \frac{\sqrt{3}}{18}(36-x)$

or $18x = 8\sqrt{3}(36-x)$ 1 m

$\Rightarrow \quad x = \frac{144\sqrt{3}}{9+4\sqrt{3}}$

Showing $\frac{d^2A}{dx^2} > 0 \Rightarrow$ Area is minimum for this value of x ½ m

$\therefore$ Length of one piece $= \frac{144\sqrt{3}}{9+4\sqrt{3}}$ cm

Length of second piece $= \frac{324}{9+4\sqrt{3}}$ cm

½ m

18. Figure 1 m

Calculating $x = -1$ and $x = 2$ as points of intersection 1 m

Required Area

$$= \int_{-1}^{2} \left[\frac{x+2}{4} - \frac{x^2}{4} \right] dx$$ 2 m

$$= \left[\frac{x^2}{8} + \frac{x}{2} - \frac{x^3}{12} \right]_{-1}^{2}$$ 1 m

$$= \frac{9}{8} \text{ sq.units}$$ 1 m

OR

$$I = \int_0^2 \left(x^2 + x\right) dx$$

Here $a = 0$, $b = 2$, $f(x) = x^2 + x$, $h = \frac{2}{n}$ 1 m

$$\therefore I = \lim_{h \to 0} h \left[f(0) + f(0+h) + f(0+2h) + \ldots + f(0+\overline{n-1}h) \right]$$ 1 m

$$= \lim_{h \to 0} h \left[0 + (h^2 + h) + (4h^2 + 2h) \ldots + \{(n-1)^2 h^2 + (n-1)h \right]$$ 1 m

$$= \lim_{h \to 0} h \left[h^2 . \frac{(n-1)\, n.(2n-1)}{6} + \frac{h.n(n-1)}{2} \right]$$ 1 m

$$= \lim_{n \to \infty} \left[\frac{8}{6n^3} (n-1)n.(2n-1) + \frac{4}{2n^2} n.(n-1) \right]$$ 1 m

$$= \frac{8}{3} + 2 = \frac{14}{3}$$ 1 m

SECTION ‘B’

19. $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}, \quad \vec{b} = 3\hat{i} + \hat{k}$

$\vec{a} = \lambda \vec{b} + \vec{c}$, where $\vec{c} \perp \vec{b}$ ½ m

$\Rightarrow \quad \vec{c} = \vec{a} - \lambda \vec{b}$ ½ m

$= (5-3\lambda)\,\hat{i} - 2\hat{j} + (5-\lambda)\hat{k}$

$\vec{c} \perp \vec{b} \quad \Rightarrow \quad \vec{b}\,.\,\vec{c} = 0$ ½ m

$\therefore \quad 3(5-3\lambda) + (-2).0 + (5-\lambda)\,1 = 0$

½ m

1 m

and $\vec{c} = -\hat{i} - 2\hat{j} + 3\hat{k}$

20. For the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ to be coplanar,

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$ 2 m

$\Rightarrow \quad c^2 - ab = 0 \quad \text{or} \quad c^2 = ab$ 1 m

21. Let $u = 18\text{km/hour} = 5$ m/sec ½ m

$200 = 2\,.\,a\,.\,500$

1 m

Again final velocity = 72km/hour = 20m/sec.

½ m

Let s be the distance travelled

$\therefore \quad 20^2 - 15^2 = \frac{2}{5} s$

or $\quad 175 = \frac{2}{5} . s \quad \Rightarrow \quad s = 437.5m$ 1 m

22. Distance travelled in nth second $= \frac{1}{2} g \,(2n - 1)$ ....(i) ½ m

Total distance covered $= \frac{1}{2} g \,.\, n^2$ ....(ii) ½ m

where n is number of seconds.

It is given that $(i) = \frac{5}{9}$ of (ii)

$$\frac{1}{2} g \,(2n - 1) = \frac{5}{9} . \frac{1}{2} g \, n^2$$ ½ m

$$5n^2 - 18n + 9 = 0$$

or $\quad n = 3, \frac{3}{5} \left[\text{neglecting } n = \frac{3}{5}\right]$ ½ m

$\therefore \quad$ Height of tower $= \frac{1}{2} g \,(3)^2 = \frac{9}{2} g \quad$ m. 1 m

OR

(i) Maximum horizontal range $= \frac{u^2}{g} = \frac{(29.4)^2}{9.8} = 88.2$ m 1½ m

(ii) Range $= \frac{u^2 \sin 2\alpha}{g} = 44.10$

$\Rightarrow \quad \sin 2\alpha = \frac{44.10 \times 9.8}{29.4 \times 29.4} = \frac{1}{2}$ 1 m

$\Rightarrow \quad 2\alpha = 30° \quad$ or $\quad \alpha = 15°$ ½ m

23. Let (α, β, γ) be the foot of perpendicular (as shown in figure)

D.R.of BC are ½ m

½ m

1 m

Equation of BC is

....(i) ½ m

As (α, β, γ) lies on (i)

$\Rightarrow \quad \alpha = \lambda, \ \beta = -\lambda - 1, \ \gamma = -2\lambda + 3$ ....(ii) 1 m

[illegible]

$\therefore$ The foot of perpendicular is ½ m

24. Resultant of P and Q acting at angle α is perpendicular to P

.........................(i) 1½ m

Similarly $S + P\cos\alpha = 0$

..........................(ii) 1½ m

From (i) and (ii)

$$\frac{P}{Q} = \frac{S}{P} \Rightarrow P^2 = Q.S \quad \text{or} \quad P = \sqrt{QS}$$ 1 m

25. Figure 1 m

In case (i), $\frac{P}{BC} = \frac{Q}{AC} = \frac{R}{AB}$

$\Rightarrow \quad BC = \frac{P}{Q-P} \cdot AB$ ....(A) 1m

In case (ii), $\frac{2P}{BC'} = \frac{Q}{AC'} = \frac{R'}{AB}$

$\Rightarrow \quad BC' = \frac{2P}{2P-Q} \cdot AB$ ....(B) 1 m

$\therefore \quad \frac{P}{Q-P} = \frac{2P}{2P-Q}$

$\Rightarrow \quad \frac{P}{Q} = \frac{3}{4}$ 1 m

OR

$\therefore \quad \frac{F_1}{\sin 135°} = \frac{F_2}{\sin 105°} = \frac{F_3}{\sin 120°}$ 1½

$\frac{F_1}{\frac{1}{\sqrt{2}}} = \frac{F_2}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{F_3}{\frac{\sqrt{3}}{2}}$ 1½ m

$\therefore \quad F_1 : F_2 : F_3 = 2 : \sqrt{3}+1 : \sqrt{6}$ 1 m

26. Cartesian equation of one-plane is $2x + 6y + 12 = 0$ or $x + 3y + 6 = 0$ ....(i) 1½ m

Equation of second plane is $3x - y + 4z = 0$ ....(ii) 1½ m

Any plane passing through the intersection of (i) and (ii) is

$x(1+3\lambda) + y(3-\lambda) + 4\lambda z + 6 = 0$ 1 m

Its distance from (0, 0, 0) is unity.

$$\Rightarrow \quad 26\lambda^2 + 10 = 36 \qquad \Rightarrow \quad \lambda = \pm 1$$ 1 m

$\therefore$ The equation of planes are

$2x + y + 2z + 3 = 0$

½ m

Their vector equation are $\vec{r}\cdot(2\hat{i} + \hat{j} + 2\hat{k}) + 3 = 0$

$\vec{r}\,.\,(\hat{i} - 2\hat{j} + 2\hat{k}) - 3 = 0$ ½ m

SECTION 'C'

19. Figure 1½ m

Maximise Z = 60x + 15y

Constraints :

$$\frac{[illegible]}{\sqrt{(1+3\lambda)^2 + (3-\lambda)^2 + 16\lambda^2}} = 1$$

Z at O = 0

Z at A = 60 × 30 + 0 = 1800

Z at B = 60 × 20 + 15 × 30 = 1650 1 m

Z at C = 15 × 50 = 750

Z is maximum when x = 30, y = 0 ½ m

20. Let the tailor A works for x days and the tailor B works for y days

We have to Minimise 150x + 200y = Z 1 m

Subject to the constraints

2 m

21. E_1, E_2 and A are the events defined as follows

E_1 : Plant I is selected to manufacture

E_2 : Plant II is selected to manufacture

A : The motor cycle is of standard quality

$P(E_1) = \frac{7}{10}$, $P(E_2) = \frac{3}{10}$, ½ m

$P(A/E_1) = \frac{8}{10}$, $P(A/E_2) = \frac{9}{10}$ ½ m

$\therefore$ By Baye's Theorem,

$$P(E_2/A) = \frac{P(E_2)\ P(A/E_2)}{P(E_1)\ P(A/E_1) + P(E_2).\ P(A/E_2)}$$ 1 m

$$= \frac{\frac{3}{10} \times \frac{9}{10}}{\frac{7}{10} \times \frac{8}{10} + \frac{3}{10} \times \frac{9}{10}}$$

$$= \frac{27}{56+27} = \frac{27}{83}$$ 1 m

22. Let p be the probalility that a student entering a university will graduate.

$\therefore$ $p = 0.4 = \frac{2}{5}$, $q = 1 - \frac{2}{5} = \frac{3}{5}$, $n = 3$ ½ m

$\therefore$ (i) P (none will graduate) $= {}^3C_0 \left(\frac{3}{5}\right)^3 = \frac{27}{125}$ 1 m

(ii) P (only one will graduate) $= {}^3C_1 \left(\frac{2}{5}\right)\left(\frac{3}{5}\right)^2 = \frac{54}{125}$ 1 m

(iii) P (All will graduate) $= {}^3C_3 \left(\frac{2}{5}\right)^3 = \frac{8}{125}$ ½ m

OR

Here $\lambda = 1$ 1 m

P (at least three misprints)

1½ m

$$= 1 - 0.3679 \times \frac{5}{2} = 0.08025$$ ½ m

23. Let the total profit be Rs x

A's Salary = 10% of Rs x $= \frac{x}{10}$ ½ m

$\therefore$ Balance of Profit

The ratio of profit sharing is 4 : 5 : 9

$$= 1 - [P(X = 0) + P(X = 1) + P(X = 2)] \quad \frac{9x}{10} \quad \frac{x}{5} \quad \frac{3x}{10} \quad e^{-1} \quad \frac{1}{1!} \quad \frac{1}{2!} \quad 1 - \frac{5}{2}e^{-1} \quad = \text{Rs } 1000000 \quad = \text{Rs } 250000$$

$\therefore$ As' share in 1 m

$\therefore$ A's total share

$$\Rightarrow \frac{3x}{10} = 300000$$

$$\Rightarrow x = 1000000 \Rightarrow \text{Total Profit}$$ 1 m

B's share 1 m

C's share = Rs 450000 ½ m

OR

Initially ratio of sharing profit = 1 : 1 ½ m

After C joins, the ratio of sharing profit = 9 : 8 : 7

$\therefore$ A has sacrificed $= \frac{1}{2} - \frac{9}{24} = \frac{1}{8}$ ½ m

B has sacrifice $= \frac{1}{2} - \frac{8}{24} = \frac{1}{6}$ ½ m

$\therefore$ Premium sharing ratio of A and B is 3 : 4 ½ m

1 m

B's share in premium = Rs (210000 − 90000) = Rs 120000 1 m

24. We know that

where P is the present value. ½ m

Here R = Rs 1200, n = 10, i = 0.12 ½ m

$\Rightarrow 150x = 7500$

$x = 50$

$P = R\left\{\frac{1-(1+i)^{-n}}{i}\right\}$

$\therefore$ A's share in premium $= \text{Rs}\left(\frac{3}{7} \times 210000\right) = \text{Rs } 90000$

$$\therefore \quad P = 1200\left\{\frac{1-(1.12)^{-10}}{0.12}\right\}$$ 1 m

$$= 1200\,\frac{(1-0.3221)}{0.12}$$ 1 m

$$= \frac{1200 \times 0.6779}{0.12} = 6779$$

$\therefore$ Present value of annuity is Rs 6779 1 m

25. (i) Here C(x) = 200x + 7500

R (x) = 350 x ½ m

For no loss, R(x) = C(x)

1 m

i.e. 50 calculators must be produced and sold daily for no loss. ½ m

(ii) Here $C(x) = 200x + 7500$

$R(x) = 500x$ ½ m

For Break even points, $R(x) = C(x)$ 1 m

or

i.e. 25 calculators be produced for break even points. ½ m

26. Let the face value of bill be Rs S, Time = 8 months

1 m

2 m

[illegible] was discounted early

$\therefore$ Srt $\left[1 - \frac{\quad}{1+rt}\right]$ Time $= \frac{\quad}{365} = \frac{\quad}{5}$, $r = 5\% = \frac{\quad}{20}$ 1 m

B.G = BD − TD

½ m

$\therefore \quad 200 = S \times \frac{1}{20} \times \frac{2}{5}\left[1 - \frac{50}{51}\right]$ ½ m

$= \frac{S}{50} \times \frac{1}{51}$

$\therefore \quad S = 200 \times 50 \times 51 = 510000$ 1 m

Note : If candidate has taken 366 days for the year and gets the answer as Rs 512770 (app.), full credit may be given.

QUESTION PAPER CODE 65/1

EXPECTED ANSWERS/VALUE POINTS

SECTION 'A'

1. $f(A) = A^2 - 2A - 3I$

$$= \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} - 2\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} - 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ ½ m

$$= \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix} + \begin{pmatrix} -2 & -4 \\ -4 & -2 \end{pmatrix} + \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$$ (1+½+½) m

$$= \begin{pmatrix} 5-2-3 & 4-4+0 \\ 4-4+0 & 5-2-3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$ ½ m

2. Using $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$ we get

$$\begin{vmatrix} 2x & -2x & 0 \\ 0 & 2x & -2x \\ a-x & a-x & a+x \end{vmatrix} = 0$$ 1 m

Using $C_2 \to C_2 + C_1$ we get

$$\begin{vmatrix} 2x & 0 & 0 \\ 0 & 2x & -2x \\ a-x & 2(a-x) & a+x \end{vmatrix} = 0$$ 1 m

$\therefore\ 2x[2x(a+x) + 4x(a-x)] = 0$ or $4x^2(3a - x) = 0 \Rightarrow x = 0, x = 3a$ 1 m

(Note : Using any two operations (1 mark each) and finding answer - 1mark)

3 Let A: the chosen integer is divisible by 6

B: the chosen integer is divisible by 8

$\therefore\ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ 1 m

(½+½+½) m

$$= \frac{50}{200} = \frac{1}{4}$$ ½ m

4. P (getting grade A in all subjects) $= (0.2)(0.3)(0.5)$

$= 0.03$ 1 m

P (getting grade A in no subject) $= (0.8)(0.7)(0.5)$

$= 0.28$ 1 m

P (getting grade A in two subjects) $= (0.8)(0.3)(0.5)+(0.2)(0.7)(0.5)+(0.2)(0.3)(0.5)$

$= 0.12+0.07+0.03 = 0.22$ 1 m

5. Writing $I = \int \frac{2\cos x.\sin x\ dx}{(a + b\cos x)^2}$ and putting $(a + b\cos x) = t$, to get $\sin x\ dx = -\frac{1}{b}dt.$ 1 m

$$\therefore\quad I = -\frac{2}{b^2}\int \frac{t-a}{t^2}\,dt = -\frac{2}{b^2}\int\left(\frac{1}{t} - \frac{a}{t^2}\right)dt$$ 1 m

$$I = \frac{1}{t}e^t + c = \frac{x}{\log x} + c$$

$$\frac{3}{200}\quad \frac{13}{200}\quad \frac{25}{200}\quad \frac{8}{200}$$

$$= -\frac{2}{b^2}\left[\log t + \frac{a}{t}\right] + c = -\frac{2}{b^2}\left[\log(a + b\cos x) + \frac{a}{a + b\cos x}\right] + C$$ 1 m

6. Put $\log x = t \Rightarrow x = e^t$ and $dx = e^t.dt$ ½ m

$$\therefore\quad I = \int\left(\frac{1}{t} - \frac{1}{t^2}\right)e^t dt$$ ½ m

using $\int [f(x) + f'(x)]e^x dx = f(x).e^x + c,$ we get ½ m

(1+½) m

7. $\frac{dy}{dx} + \cot x.y = x^2\cot x + 2x,\quad I.F = e^{\int \cot x dx} = \sin x$ 1 m

$$\therefore\quad y.\sin x = \int (x^2.\cos x + 2x\sin x)dx$$ ½ m

$\therefore \quad y.\sin x = x^2.\sin x - \int 2x \sin x \, dx + \int 2x \sin x \, dx + c$ 1 m

$\Rightarrow \quad y \sin x = x^2 \sin x + c \text{ or } y = x^2 + c.\operatorname{cosec} x$ ½ m

8. Here, $\frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy}$, Put $\frac{y}{x} = \nu \quad \therefore \frac{dy}{dx} = \nu + x\frac{d\nu}{dx}$ ½ m

$\therefore \quad \nu + x\frac{d\nu}{dx} = \frac{1+\nu^2}{1+\nu} \quad \text{or} \quad x\frac{d\nu}{dx} = \frac{1-\nu}{1+\nu}$ ½ m

$\therefore \quad \int \frac{\nu+1}{\nu-1} d\nu = -\int \frac{dx}{x} \qquad \therefore \quad \int \left(1 + \frac{2}{\nu - 1}\right) d\nu = -\int \frac{dx}{x}$ (½ +½) m

$\therefore \quad \nu + 2\log|\nu - 1| + \log x = c. \quad \therefore \quad y + 2x\log\left|\frac{y-x}{x}\right| + x\log x = cx$ (½ +½) m

OR

$\frac{d^2y}{dx^2} = e^x + \cos x \qquad \Rightarrow \quad \frac{dy}{dx} = e^x + \sin x + C_1$ ½ m

$\frac{dy}{dx} = 1 \quad \text{when} \quad x = 0 \Rightarrow C_1 = 0$ ½ m

$\therefore \quad \frac{dy}{dx} = e^x + \sin x \qquad \Rightarrow \quad y = e^x - \cos x + C_2$ ½ m

$y = 1 \quad \text{when} \quad x = 0 \quad \Rightarrow \quad C_2 = 1$ ½ m

$\therefore \quad y = e^x - \cos x + 1$ 1 m

9. The truth table is

		Hypotheses		Conclusion		
p	q	pvq	~ p	~ q		
T	T	T	F	F	FOR TRUTH TABLE :	2½ m
T	F	T	F	T		
F	T	T	T	F	← Critical row Identification :	1 m
F	F	F	T	T		

There is only one critical row in which the conclusion is false. ½ m

Hence, the given argument is invalid.

OR

LHS 1 m

1 m

$= \ y + (x + y') \ = x + (y + y')$ 1 m

1 m

10. ½ m

$$= \lim_{y \to 0} \frac{x[\cos x - \cos (x+y)]}{y.\cos x.\cos (x+y)} + \sec x$$ 1 m

$$= \lim_{y \to 0} \frac{x\left[2 \sin\left(\frac{2x+y}{2}\right) \sin\left(\frac{y}{2}\right)\right]}{2.\cos x.\cos (x+y)\frac{y}{2}} + \sec x$$ 1 m

$= y (x + y) \sec (x + y) + x \sec x \ldots$ $f(x) = \tan\sqrt{x}$ $\therefore$ $f'(x) = \lim_{\Delta x \to 0}$ $= \lim_{y \to 0}$ $[\sec (x+y) - \sec x]$ y Δx ½ m

$= \ x.\tan x.\sec x + \sec x \quad$ or $\quad \sec x\ (x \tan x + 1)$

11. Let ½ m

$$\therefore \ f'(x) = \lim_{\Delta x \to 0} \frac{\tan\left[\sqrt{x + \Delta x} - \sqrt{x}\right]\left[1 + \tan\sqrt{x}\tan\sqrt{x+\Delta x}\right]}{\Delta x}$$ 1 m

$$= \lim_{\Delta x \to 0} \frac{\tan\left[\sqrt{x+\Delta x} - \sqrt{x}\right]}{\sqrt{x+\Delta x} - \sqrt{x}}.\left[1 + \tan\sqrt{x}.\tan\sqrt{x+\Delta x}\right]\frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}$$ 1 m

$$= \left[1 + \tan^2\sqrt{x}\right]\lim_{\Delta x \to 0}\left(\frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}.\frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}\right)$$ 1 m

$$= \ \sec^2\sqrt{x}.\frac{1}{2\sqrt{x}}$$ ½ m

12. $y = \left[x + \sqrt{x^2 + a^2}\right]^n \Rightarrow \dfrac{dy}{dx} = n \,.\left[x + \sqrt{x^2 + a^2}\right]^{n-1} . \left[1 + \dfrac{2x}{2\sqrt{x^2 + a^2}}\right]$ 1 m

$\therefore \quad \dfrac{dy}{dx} = n.\left[x + \sqrt{x^2 + a^2}\right]^{n-1} . \dfrac{\left(x + \sqrt{x^2 + a^2}\right)}{\sqrt{x^2 + a^2}}$ 1 m

$= n.\left[x + \sqrt{x^2 + a^2}\right]^n . \dfrac{1}{\sqrt{x^2 + a^2}}$ 1 m

$= \dfrac{ny}{\sqrt{x^2 + a^2}}$ 1 m

13. $f(x) = 2x^3 - 15x^2 + 36x + 1 \Rightarrow f'(x) = 6x^2 - 30x + 36$ ½ m

$f'(x) = 0 \Rightarrow 6(x^2 - 5x + 6) = 0 \Rightarrow (x-3)(x-2) = 0, \Rightarrow x = 3, x = 2$ ½ m

Possible intervals are $(-\infty, 2), (2, 3), (3, \infty)$ ½ m

for $f(x)$ increasing in $(-\infty, 2) \cup (3, \infty)$ and 1 m

decreasing in $(2, 3)$ ½ m

Points at which the tangents are parallel to x-axis are $(3, 28), (2, 29)$ 1 m

14. $I = \displaystyle\int \frac{x^2}{x^2 + 6x + 12}\, dx = \int \left(1 - \frac{6x + 12}{x^2 + 6x + 12}\right) dx$ 1 m

$= x - 3\displaystyle\int \frac{2x + 6 - 2}{x^2 + 6x + 12}\, dx = x - 3\int \frac{2x + 6}{x^2 + 6x + 12}\, dx + 6\int \frac{1}{x^2 + 6x + 12}\, dx$ 1 m

$= x - 3.\log\left|x^2 + 6x + 12\right| + 6\displaystyle\int \frac{1}{(x+3)^2 + \left(\sqrt{3}\right)^2}\, dx$ 1 m

$= x - 3.\log\left|x^2 + 6x + 12\right| + \dfrac{6}{\sqrt{3}} \tan^{-1}\left(\dfrac{x+3}{\sqrt{3}}\right) + C$ 1 m

15. $\int_{-5}^{0} f(x)dx = \int_{-5}^{0} |x|dx + \int_{-5}^{0} |x+2|dx + \int_{-5}^{0} |x+5|dx$ ½ m

$= \int_{-5}^{0} -(x)dx + \int_{-5}^{-2} -(x+2)dx + \int_{-2}^{0} (x+2)dx + \int_{-5}^{0} (x+5)dx$ 2 m

$= \left[-\frac{x^2}{2}\right]_{-5}^{0} + \left[-\frac{x^2}{2} - 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{0} + \left[\frac{x^2}{2} + 5x\right]_{-5}^{0}$ ½ m

$= \frac{25}{2} + \left[(-2+4) - \left(\frac{-25}{2} + 10\right)\right] + \left[0 - (2-4)\right] + 0 - \left(\frac{25}{2} - 25\right)$

$= \frac{25}{2} + 2 + \frac{5}{2} + 2 + \frac{25}{2} = \frac{63}{2}$ 1 m

16. Figure 1 m

Let O be the centre of sphere $\therefore$ OC = 12cm

Let radius of cone = x cm and height = h cm

$x^2 + (h-12)^2 = 144$ and

h = AD = AO+OD=12+OD OD = (h – 12)cm

1 m

Volume of cone $= \frac{1}{3}\pi x^2.h = \frac{1}{3}\pi h\left[144 - (h-12)^2\right]$ ½ m

$V = \frac{\pi}{3}\left[144h - h^3 + 24h^2 - 144h\right]$

$V = \frac{\pi}{3}\left[24h^2 - h^3\right]$ 1 m

$\frac{dv}{dh} = 0 \Rightarrow 48h - 3h^2 = 0, \Rightarrow 16 - h = 0 \Rightarrow h = 16$ cm 1 m

$\frac{d^2v}{dh^2} = \frac{\pi}{3}\left[48 - 6h\right] = \pi(-48)$ i.e. negative 1 m

$\therefore$ For Maximum volume, height = 16cm ½ m

OR

Getting the point of intersection 1 m

$x = y^2 \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2y} \quad \therefore \quad \text{Slope } (m_1) = \dfrac{1}{2k^{1/3}}$ 1½ m

$xy = k \Rightarrow \dfrac{dy}{dx} = \dfrac{-y}{x} \quad \therefore \quad \text{Slope } (m_2) = -\dfrac{1}{k^{1/3}}$ 1½ m

$m_1.m_2 = -1 \Rightarrow \dfrac{1}{2k^{2/3}} = 1$ 1 m

$2k^{2/3} = 1 \Rightarrow 8k^2 = 1$ 1 m

17. Writing the given equations as

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & -1 \\ 3 & 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \text{ or } A.X = B \Rightarrow X = A^{-1}.B$$ 1 m

$\left(k^{2/3} = 3k^{1/3}\right)$

$|A| = 1(3) - 1(1) - 1(4) = 3 - 1 - 4 = -2$ 1 m

$a_{11} = 3, \quad a_{12} = -1 \quad a_{13} = 4$

$a_{21} = 1 \quad a_{22} = 1 \quad a_{23} = 2$

$a_{31} = -2 \quad a_{32} = 0 \quad a_{33} = -2$

[**Note :** For every four correct cofactors, one mark may be given] 2 m

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 3 & 1 & -2 \\ -1 & 1 & 0 \\ 4 & 2 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$ ½ m

$$= \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

$\therefore \quad x = 2, \; y = 1, \; z = 2$ 1½ m

18. Correct figure 1 m

Point of intersection of two curves is at $x = 1$ 1 m

Required area (1+1) m

$$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$ 1 m

$$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \text{ sq. units}$$ 1 m

OR

Correct figure 1 m

Getting $y = -2$ and $y = 1$ as points of intersection 1 m

Required area $= \int_{-2}^{1}(2-y)dy - \int_{-2}^{1} y^2 dy$ 2 m

$$= \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]_{-2}^{1}$$ 1 m

$$\left.\begin{aligned} &= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right) \\ &= \frac{9}{2} \text{ sq. units} \end{aligned}\right\}$$ 1 m

SECTION 'B'

19. Given that $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$ and

$$\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$$ 1 m

Let θ be the angle between and $\left(\vec{a} + \vec{b} + \vec{c}\right)$

$$\therefore \quad \cos\theta = \frac{\vec{a}\cdot\left(\vec{a}+\vec{b}+\vec{c}\right)}{|\vec{a}|\,|\vec{a}+\vec{b}+\vec{c}|} = \frac{|\vec{a}|^2 + \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}}{|\vec{a}|\,|\vec{a}+\vec{b}+\vec{c}|}$$ 1 m

$$\therefore \quad \cos\theta = \frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|} \quad \Rightarrow \quad \theta = \cos^{-1}\frac{1}{\sqrt{3}}$$ 1 m

20. A, B, C, D are coplaner if $\left[\vec{AB}, \vec{AC}, \vec{AD}\right] = 0$ ½ m

or Scalar triple product of any three vectors through all four points = 0

$$\left.\begin{aligned} \vec{AB} &= 10\hat{i} - 12\hat{j} - 4\hat{k}, \qquad \vec{AC} = -6\hat{i} + 10\hat{j} - 6\hat{k} \\ \vec{AD} &= -4\hat{i} + 2\hat{j} + 10\hat{k} \end{aligned}\right\}$$ 1½ m

$$\therefore \quad \left[\vec{AB}, \vec{AC}, \vec{AD}\right] = \begin{bmatrix} 10 & -12 & -4 \\ -6 & 10 & -6 \\ -4 & 2 & 10 \end{bmatrix}$$ ½ m

$\therefore$ vt = 126 $\Rightarrow$ 3v = 126 $\Rightarrow$ v = 42 m/s

using $v = u + at$ we get $42 = u + 3a$

using $s = ut + \frac{1}{2}at^2$ we get $108 = 3u + \frac{9}{2}a$

$$= 10\,(112) + 12(-84) - 4(-12 + 40) = 1120 - 1008 - 112 = 0$$ ½ m

21.

A	108 m	B	126 m	C
	t = 3 s.	v	t = 3 s.	

Let velocity at an initial point = u m/s, and it covers a distance of 108m in 3 sec. with acceleration a m/s^2 to reach B with velocity v m/s. and in next 3sec. it covers 126m in 3 seconds without any accelaration.

1 m

.................................(i) and ½ m

.....................(ii) ½ m

Solving (i) and (ii) we get $u = 30$ m/s and $a = 4$ m/s^2 1 m

22. $u = 24$ m/s $\qquad \alpha = 60°$

(i) Equation of path is

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \qquad \therefore \quad y = \sqrt{3}\,x - \frac{gx^2}{2\times 576\times\frac{1}{4}}$$

$$\text{or} \quad \sqrt{3}\,x - y = \frac{1}{288}g\,x^2$$

(½ +½) m

(ii) $T = \dfrac{2u\sin\alpha}{g} = \dfrac{2\times 24\times\frac{\sqrt{3}}{2}}{9.8} = 4.24$ sec (½+½) m

(iii) Maximum height $= \dfrac{u^2\sin^2\alpha}{2g} = \dfrac{576\times\frac{3}{4}}{19.6} = 22.04$ m (½+½) m

OR

$\left(\hat{i}+2\hat{j}+3\hat{k}\right)\times\left(-3\hat{i}+2\hat{j}+5\hat{k}\right)$

At A (15, 10), we can write

$$10 = 15\tan\alpha - \frac{225g}{2u^2\cos^2\alpha} \text{ ....(i)}$$ 1 m

At B (45, 10)

$$10 = 45\tan\alpha - \frac{2025g}{2u^2\cos^2\alpha} \text{ ....(ii)}$$ 1 m

Multiplying (i) by 9 and subtracting from (ii) we get

$$-80 = -90\tan\alpha \quad \Rightarrow \quad \tan\alpha = \frac{8}{9} \qquad \therefore \quad \alpha = \tan^{-1}\frac{8}{9}$$ 1 m

23. A vector $\vec{m}$ perpendicular to both given lines is 1 m

$$\text{i.e.} \quad \vec{m} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -3 & 2 & 5 \end{vmatrix} = 4\hat{i} - 14\hat{j} + 8\hat{k} \quad \text{or} \quad 2\hat{i} - 7\hat{j} + 4\hat{k}$$ 1½ m

Equation of line passing through and parallel to is

1½ m

24. If is the angle between then

$R^2 = P^2 + Q^2 + 2PQ \cos\alpha$ ....(i) ½ m

$4R^2 = P^2 + 4Q^2 + 4PQ \cos\alpha$ ....(ii) and ½ m

$4R^2 = P^2 + Q^2 - 2PQ \cos\alpha$ ....(iii) ½ m

Eliminating α between (i) and (ii) we get ½ m

Eliminating α between (i) and (iii) we get ½ m

$\therefore \quad \frac{P^2}{6} = \frac{Q^2}{9} = \frac{R^2}{6} \quad \text{or} \quad \frac{P^2}{2} = \frac{Q^2}{3} = \frac{R^2}{2}$ 1 m

[illegible]

$\therefore \quad P : Q : R = \sqrt{2} : \sqrt{3} : \sqrt{2}$ ½ m

OR

Correct figure ½ m

Since $(30)^2 + (40)^2 = (50)^2 \quad \therefore \quad \angle ACB = 90°$ ½ m

$\therefore \quad \angle ACD = (90 + A)°$ and $\angle BCD = (90 + B)°$ ½ m

Using Lami's theorem we get

$$\frac{T_1}{\sin(90° + B)} = \frac{T_2}{\sin(90° + A)} = \frac{W}{\sin 90°} \Rightarrow \frac{T_1}{\cos B} = \frac{T_2}{\cos A} = \frac{W}{1}$$ 1 m

$\therefore \quad T_1 = W \cos B = W.\left(\frac{40}{50}\right) = \frac{4}{5} W$ ½ m

$T_2 = W \cos A = W.\left(\frac{30}{50}\right) = \frac{3}{5}W$ ½ m

$\therefore \quad T_1 : T_2 = 4 : 3$ ½ m

25. Figure ½ m

Let a force of 18N act at A and of 10N at B and the resultant at C. Let AB = x cm ½ m

$\therefore \frac{18}{BC} = \frac{10}{AC} = \frac{8}{AB}$ or $\frac{18}{12} = \frac{10}{12-x} = \frac{8}{x}$ (1+1) m

$\Rightarrow \quad x = \frac{16}{3}$ ½ m

$\therefore$ Distance between the lines of action of two forces

½ m

26. Let the equation of sphere be $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ ....(i) [illegible]

$(1, -3, 4)$ lies on sphere $\therefore 1 + 9 + 16 + 2u - 6v + 8w + d = 0$

....................................(ii) 1 m

..................................(iii) 1 m

....(iv) 1 m

....(v) ½ m

2 m

½ m

SECTION 'C'

19. Let E_1 : getting 5 when a die is tossed. E_2 : Not getting a 5

H : reports that it was 5

$\therefore \quad P(E_1) = \frac{1}{6} \qquad P(E_2) = \frac{5}{6}$ ½ m

$P(H/E_1) = \frac{8}{10} = \frac{4}{5} \qquad P(H/E_2) = \frac{1}{5}$ ½ m

$\therefore \quad P(E_1/H) = \frac{P(E_1)P(H/E_1)}{P(E_1)P(H/E_1) + P(E_2)P(H/E_2)}$ 1 m

$= \frac{\frac{1}{6}.\frac{4}{5}}{\frac{1}{6}.\frac{4}{5} + \frac{5}{6}.\frac{1}{5}} = \frac{4}{9}$ 1 m

20. Here n = 4 probability of success $(p) = \frac{1}{2}$

probability of failure $(q) = \frac{1}{2}$

$\therefore \quad P(X = 0) = e^{-2} = 0.13534$ 1 m

Since distribution is binomial $\therefore$ Mean $= np = 4.\frac{1}{2} = 2$ 1 m

Variance $= npq = 4.\frac{1}{2}.\frac{1}{2} = 1$ 1 m

OR

$P(X = 1) = P(X = 2) \Rightarrow \frac{e^{-\lambda}.\lambda^1}{1!} = \frac{e^{-\lambda}.\lambda^2}{2!}$ (½+½) m

$\therefore$ Mean of the distribution $(\lambda) = 2$ ½ m

½ m

$P(X = 4) = \frac{e^{-2}(2)^4}{4!} = \frac{16}{24}(0.13534) = 0.09023$ 1 m

21. $\text{B.G.} = \frac{1}{9}\text{BD} \Rightarrow (\text{B.D} - \text{TD}) = \frac{1}{9}.\text{BD} \Rightarrow 8\text{B.D} = 9\text{T.D}$ 1 m

$\Rightarrow 8.\text{S.r.t} = 9.\frac{\text{S.r.t}}{1+\text{rt}} \Rightarrow 8\ \text{rt} = 1$ 1 m

$t = \frac{1}{8} \times \frac{100}{10} = \frac{5}{4} = 1.25$ years. 1 m

22. S = Rs 5300, Legal due date is 19th Sept. 2003.

$\therefore$ From 12th February to 19th September, number of days

= (16+31+30+31+30 +31+31+19) = 219 days. 1 m

$\therefore$ Discounted value $= \text{Rs}\left[5300 - 318\right] = \text{Rs } 4982$ 1 m

$\text{B.G} = \text{BD} - \text{TD} = 318 - \dfrac{318}{1+\frac{1}{10}\times\frac{3}{5}} = 318\left[1 - \frac{50}{53}\right] = \text{Rs } 18$

$\text{B.D} = \text{S.r.t} = 5300 \times \frac{10}{100} \times \frac{219}{365} = \text{Rs } 318$ 1 m

23. Let total investment be Rs x and total time be 't' months.

$\therefore$ A invested Rs $\frac{x}{2}$ for a period of $\frac{t}{2}$ months

$\therefore$ A's adjusted capital for 1 month $= \text{Rs } \frac{x}{2}.\frac{t}{2}$ ½ m

Similarly B's adjusted capital for 1 month $= \text{Rs } \frac{x}{3}.\frac{t}{3}$ ½ m

and C's adjusted capital for 1 month $= \text{Rs } \frac{x}{6}.t$ ½ m

$\therefore$ Ratio is $\frac{1}{4} : \frac{1}{9} : \frac{1}{6}$ or $9 : 4 : 6$ 1 m

$\therefore \quad A\text{'s share in Profit} = \text{Rs } 1{,}90{,}000 \times \frac{9}{19} = \text{Rs } 90{,}000$ ½ m

$B\text{'s share in Profit} = \text{Rs } 1{,}90{,}000 \times \frac{4}{19} = \text{Rs } 40{,}000$ ½ m

$\therefore \quad C\text{'s share} = 60{,}000$ ½ m

24. A = Rs 5,52,000 n = 5 i = 0.05 1 m

Using 1 m

$$\therefore \quad P = \text{Rs } \frac{552000 \times 0.05}{(1.05)^5 - 1} = \text{Rs } \frac{5520 \times 5}{1.276 - 1} = \text{Rs } \frac{5520 \times 5}{0.276}$$ 1½ m

$= \text{Rs } 1{,}00{,}000$ ½ m

25.

$$A = \frac{P}{i}\left[(1+i)^n - 1\right] \qquad P = \frac{A.i}{\left[1+i\right]^n - 1}$$

$$C(x) = 300x - 10x^2 + \frac{1}{3}x^3 \qquad AC = 300 - 10x + \frac{1}{3}x^2$$

(i) $M.C = C'(x) = 300 - 20x + x^2$ ½ m

$\therefore \quad \frac{d}{dx}(MC) = 0 \Rightarrow -20 + 2x = 0 \Rightarrow x = 10$ units 1 m

and $\frac{d^2}{dx^2}(MC) = 2$ i.e + ve

½ m

$\therefore$ Output for minimum MC = 10 units.

(ii) ½ m

$M.C = AC \Rightarrow 300 - 20x + x^2 = 300 - 10x + \frac{1}{3}x^2$ 1 m

$\frac{2}{3}x^2 - 10x = 0 \Rightarrow x = 15$ units

½ m

$\therefore$ Output at which MC = AC is 15 units

OR

(i) At Break even point C(x) = R(x)

$\therefore \quad 5x + 350 = 50x - x^2 \quad \text{or} \quad x^2 - 45x + 350 = 0$ 1 m

$\Rightarrow \quad x = 10 \text{ units} \quad \text{or} \quad x = 35 \text{ units}$ 1 m

(ii) ½ m

or $x^2 - 45x + 350 > 0, \quad (x - 35)(x - 10) > 0$ ½ m

$\Rightarrow \quad x > 35 \quad \text{or} \quad x < 10$

So, there will be loss if firm produces less than 10 units ½ m

Or more than 35 units. ½ m

26. If x number of trunks of Ist type & y number of trunks of second type be manufactured.

Maximise P = 30x + 25y ½ m

[illegible] For loss $R(x) - C(x) < 0 \quad 50x - x^2 - 5x - 350 < 0$

Subject to : 1 m

1 m

Correct graph 2 m

Extreme points of feasible region are

A(0, 6), B(3, 3) and C(5, 0) ½ m

Profit at A = 30 × 0 + 25 × 6 = Rs. 150

Profit at B = 30 × 3 + 25 × 3 = Rs. 165

Profit at C = 30 × 5 + 25 × 0 = Rs. 150

For maximum profit he should manufacture 3 trunks of each type. 1m