S.E (COMPUTER INFORMATION TECHNOLOGY) (SEM III) (REV)
EXAMINATION OCTOBER, 2006
Discrete Structures

[REVISED COURSE]

CON/5180-06. YM-5242

5/06/06

(3 Hours) [Total Marks : 100

N.B.:

1) Question number 1 is compulsory.
2) Attempt any four questions out of remaining six questions.
3) Assumptions made should be clearly stated.
4) Figures to the right indicate full marks.
5) Assume suitable data wherever required but justify the same.

Q. No.1 a) (i) How many numbers must be selected from the set {1,2,3,4,5,6} to Guarantee that at least one pair of these numbers add up to 7? (05)

(ii) Use mathematical induction to prove the following inequality. (05)
$n < 2^n$ for all positive integers n.

b) (i) Find greatest lower bound and least upper bound of the set {3,9,12} and { 1,2,4,5,10} if they exist in the poset (Z^+, /). (05)

(ii) Find all solutions of the recurrence relation (05)
$a_n = 5a_{n-1} - 6a_{n-2} + 7^n$

Q. No.2 a) (i) Draw the Hasse diagram of D_{36}. (05)

(ii) Is the poset A = {2,3,6,12,24,36,72} under relation of divisibility a Lattice? Justify your answer. (05)

b) (i) Let m be the positive integer greater than 1. Show that the relation $R = \{(a,b) \mid a \equiv b \pmod m\}$ i,e aRb if and only if m divides a-b is equivalence relation on the set of integers. (05)

(ii) Show that the set $R = \{x \mid x = a+b\sqrt{2}$, a and b are integers } is a ring with ordinary addition and multiplication. (05)

Q. No.3 a) (i) Determine the generating function of the numeric function a_r where (05)
i) $a_r = 3^r + 4^{r+1}, r \geq 0$
ii) $a_r = 5, r \geq 0$

(ii) Prove that if (F, + , .) is field then it is an integral domain. (05)

b) (i) Find how many integers between 1 and 60 are not divisible by 2, nor by 3 and nor by 5 ? (05)

(ii) Let f: A→B be one to one and onto then prove that (05)
$f^{-1} \circ f = I_A$
$f \circ f^{-1} = I_B$
Where I_A and I_B are identical mapping on set a and set B.

Q. No.4 a) (i) Consider Z together with binary operations of ⊕ and ⊙ which are defined by (05)
$x \oplus y = x + y - 1$ and $x \odot y = x + y - xy$
then prove that ($Z \oplus \odot$) is an integral domain.

(ii) Define planar graph. What are the necessary and sufficient Conditions to exist Euler path, Euler circuit and Hamiltonian Circuit? (05)

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b) (i) Let m = 2, n = 5 and (05)

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Determine the group code $e_H = B^2 \rightarrow B^5$

(ii) In any Ring (R + .) prove that (05)

i) The zero element z is unique.

ii) The additive inverse of each ring element is unique.

Q. No.5 a) (i) If S is nonempty set. Prove that the set P(S) (power set of S), where $A * B = A \oplus B$ (symmetric difference of A and B) is abelian group. (05)

(ii) Prove that (Use laws of logic) (05)

$((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R)) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is tautology.

b) (i) Let A = {1,2,3,4} and let R = { (1,2) (2,3) ((3,4(2,1)} . Find transitive Closure of R using warshall's algorithm. (05)

(ii) Show that if $f : G \rightarrow G'$ is an isomorphism then $f^{-1} : G' \rightarrow G$ is also an isomorphism. (05)

No.6 a) (i) Give the exponential generating functions for the sequences given bellow (05)

i) {1,1,1, -------} ii) {0,1,0,-1,0,1,0,-1,-----}

(ii) If f is a homomorphism from a commutative semigroup (S, *) onto a Semigroup (T, *'). Then prove that (T, *') is also commutative. (05)

b)(i) Prove that (05)

A X B = (A X B) U (A X C)

ii) Consider chains of divisors of 4 and 9 i,e $L_1 = \{1,2,4\}$ and $L_2 = \{1,3,9\}$ and partial ordering relation of division on L_1 and L_2 Draw the lattice $L_1 \times L_2$. (05)

No.7 a) (i) R = {0,2,4,6,8}. Show that R is a commutative ring under addition and multiplication modulo 10. Verify whether it is field or not. (05)

(ii) Let L be the bounded distributive lattice. Prove that if complement exist Then it is unique. (05)

b)(i) Show that (2,5) encoding function $e: B^2 \rightarrow B^5$ defined by (05)

e (00) = 00000

e (01) = 01110

e (10) = 10101

e (11) = 11011

is a group code.

(ii) Let R be the relation on set A then prove that if R is symmetric then R^{-1} and $\overline{R}$ is also symmetric. (05)