Test Code MS (Short answer type) 2012

Syllabus for Mathematics

Combinatorics; Elements of set theory. Permutations and combinations. Binomial and multinomial theorem. Theory of equations. Inequalities.

Linear Algebra: Vectors and vector spacs. Matrices. Determinants. Solution of linear equations. Trigonometry. Co-ordinate geometry.

Complex Numbers: Geometry of complex numbers and De Moivres theorem.

Calculus: Convergence of sequences and series. Functions. Limits and continuity of functions of one or more variables. Power series. Differentiation. Leibnitz formula. Applications of differential calculus, maxima and minima. Taylor's theorem. Differentiation of functions of several variables. Indefinite integral. Fundamental theorem of calculus. Riemann integration and properties. Improper integrals. Double and multiple integrals and applications.

Syllabus for Statistics and Probability

Probability and Sampling Distributions: Notions of sample space and probability. Combinatorial probability. Conditional probability and independence. Random variables and expectations. Moments and moment generating functions. Standard univariate discrete and continuous distributions. Joint probability distributions. Multinomial distribution. Bivariate normal and multivariate normal distributions. Sampling distributions of statistics. Weak law of large numbers. Central limit theorem.

Descriptive Statistics: Descriptive statistical measures. Contingency tables and measures of association. Product moment and other types of correlation. Partial and multiple correlation. Simple and multiple linear regression.

Statistical Inference: Elementary theory of estimation (unbiasedness, minimum variance, sufficiency). Methods of estimation (maximum likelihood method, method of moments). Tests of hypotheses (basic concepts and simple applications of Neyman-Pearson Lemma). Confidence intervals. Inference related to regression. ANOVA. Elements of nonparametric inference.

Design of Experiments and Sample Surveys: Basic designs such as CRD, RBD, LSD and their analyses. Elements of factorial designs. Conventional sampling techniques (SRSWR/SRSWOR) including stratification. Ratio and regression methods of estimation.

Sample Questions

1. Find a basis for the subspace of $\mathbb{R}^4$ spanned by the four vectors

$$\begin{pmatrix}1\\1\\2\\4\end{pmatrix}, \begin{pmatrix}2\\-1\\-5\\2\end{pmatrix}, \begin{pmatrix}1\\1\\-4\\0\end{pmatrix}, \begin{pmatrix}2\\1\\1\\6\end{pmatrix}.$$

2. Let f be a polynomial. Assume that $f(0) = 1$, $\lim_{x\to\infty} f''(x) = 4$ and $f(x) \geq f(1)$ for all $x \in \mathbb{R}$. Find $f(2)$.

3. Let X_1 and X_2 be i.i.d. exponential random variables with mean $\lambda > 0$. Let $Y_1 = X_1 - X_2$ and $Y_2 = RX_1 - (1 - R)X_2$, where R is a Bernoulli random variable with parameter $1/2$ and is independent of X_1 and X_2.

   (a) Show that Y_1 and Y_2 have the same distribution.

   (b) Obtain the common density function.

4. Let $\begin{pmatrix}X\\Y\end{pmatrix}$ be a bivariate normal vector such that $\mathbb{E}(X) = \mathbb{E}(Y) = 0$ and $\mathrm{Var}(X) = \mathrm{Var}(Y) = 1$. Let $S \subset \mathbb{R}^2$ be defined by

$$S = \{(a, b) : aX + bY \text{ is independent of } Y\}.$$

   (a) Show that S is a subspace.

   (b) Find its dimension.

5. Let $X_1, X_2, \ldots, X_j \ldots$ be i.i.d. $N(0, 1)$ random variables. Show that for any $a > 0$,

$$\lim_{n\to\infty} \mathbb{P}\left(\sum_{i=1}^{n} X_i^2 \leq a\right) = 0.$$

6. There are two biased coins – one which has probability $1/4$ of showing heads and $3/4$ of showing tails, while the other has probability $3/4$ of showing heads and $1/4$ of showing tails when tossed. One of the two coins is chosen at random and is then tossed 8 times.

(a) Given that the first toss shows heads, what is the probability that in the next 7 tosses there will be exactly 6 heads and 1 tail?

(b) Given that the first toss shows heads and the second toss shows tail, what is the probability that the next 6 tosses all show heads?

7. Consider a randomized (complete) block design with 4 treatments and 5 replications and, let t_i be the effect of the i-th treatment ($1 \leq i \leq 4$). Consider the following three treatment contrasts.

$$\frac{1}{\sqrt{2}}(t_1 - t_2), \quad \frac{1}{\sqrt{6}}(t_1 + t_2 - 2t_3) \quad \text{and} \quad \frac{1}{\sqrt{12}}(t_1 + t_2 + t_3 - 3t_4).$$

(a) Find the variances of the best linear unbiased estimators of the above treatment contrasts.

(b) Find all the covariances between them.

8. Let V_1 be the variance of the estimated mean from a stratified random sample of size n with proportional allocation. Assume that the strata sizes are such that the allocations are all integers.

Let V_2 be the variance of the estimated mean from a simple random sample of size n.

Show that the ratio V_1/V_2 is independent of n.

9. Suppose X_1 and X_2 are i.i.d. Bernoulli random variables with parameter p where it is known that $\frac{1}{3} \leq p \leq \frac{2}{3}$. Find the maximum likelihood estimator $\widehat{p}$ of p based on X_1 and X_2.

10. Let $X_1, X_2, \ldots, X_{10}$ be i.i.d. Poisson random variables with unknown parameter $\lambda > 0$. Find the minimum variance unbiased estimator of $\exp\{-2\lambda\}$.

Note: For more sample questions you can visit
`http://www.isical.ac.in/~deanweb/MSTATSQ.html`.