Test Booklet No.
प्रश्नपत्रिका क्र.

M

Paper-III
MATHEMATICAL SCIENCE

Signature and Name of Invigilator

1. (Signature) ....

(Name) ....

2. (Signature) ....

(Name) ....

Seat No. | | | | | | |

(In figures as in Admit Card)

Seat No. ....

(In words)

OMR Sheet No. | | | | | | |

(To be filled by the Candidate)

DEC - 30313

Time Allowed : 2½ Hours] **[Maximum Marks : 150**

Number of Pages in this Booklet : **40** Number of Questions in this Booklet : **75**

Instructions for the Candidates

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. This paper consists of **75** objective type questions. Each question will carry *two* marks. *All* questions of Paper-III will be compulsory, covering entire syllabus (including all electives, without options).
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (*i*) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   (*ii*) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/ questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.**
   (*iii*) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : where (C) is the correct response.
   (A) (B) ● (D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table, etc., is prohibited.**
12. **There is no negative marking for incorrect answers.**

विद्यार्थ्यांसाठी महत्त्वाच्या सूचना

1. परिक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. सदर प्रश्नपत्रिकेत 75 बहुपर्यायी प्रश्न आहेत. प्रत्येक प्रश्नास **दोन** गुण आहेत. या प्रश्नपत्रिकेतील **सर्व** प्रश्न सोडविणे अनिवार्य आहे. सदरचे प्रश्न हे या विषयाच्या संपूर्ण अभ्यासक्रमावर आधारित आहेत.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनीटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात.
   (*i*) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्विकारू नये.
   (*ii*) **पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चूकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.**
   (*iii*) वरीलप्रमाणे सर्व पडताळून पहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   उदा. : जर (C) हे योग्य उत्तर असेल तर.
   (A) (B) ● (D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहीलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणा व्यतिरीक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापी, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. **फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.**
11. **कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.**
12. **चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.**

MATHEMATICAL SCIENCE

Paper III

Time Allowed : 2½ Hours] **[Maximum Marks : 150**

Note : This Paper contains **Seventy Five (75)** multiple choice questions. Each question carries **Two (2)** marks. Attempt *All* questions.

1. Which of the following is *true* ?

   A subset of A of $\mathbf{R}^n$ is compact if :

   (A) the complement of A is an open set

   (B) A is connected and closed

   (C) A is closed and bounded

   (D) every limit point of A is in A

   Or

   The cubic polynomial $y(x)$ which takes the following values is :

x	$y\ (x)$
1	24
3	120
5	336
7	720

   (A) $Y(x) = x^3 + x^2 + 11x + 6$

   (B) $Y(x) = 6x^3 + x^2 + 11x + 6$

   (C) $Y(x) = x^3 + 6x^2 + 11x + 6$

   (D) $Y(x) = 2x^2 + 6x^2 + 11x + 6$

2. Let f be a continuous function on $[-\pi, \pi]$ with $f(-\pi) = f(\pi)$ and suppose that f has a bounded piecewise continuous derivative on $[-\pi, \pi]$. Let the Fourier series for $f(x)$ be

   $$\frac{a_0}{2} + \sum_{k=0}^{\infty} (a_k \cos kx + b_k \sin kx).$$

   Consider the following statements :

   (1) The Fourier series for $f(x)$ converges to $f(x)$ uniformly

   (2) $a_n \to 0$ and $b_n \to 0$ as $n \to \infty$

   (3) $\sum_{n=0}^{\infty} |a_n| < \infty$ and

   $\sum_{n=1}^{\infty} |b_n| < \infty$

   Then :

   (A) Only (1) and (2) are correct

   (B) Only (2) and (3) are correct

   (C) Only (1) and (3) are correct

   (D) All are correct

Or

The Newton's forward difference interpolation formula is useful for :

(A) Extrapolation near the beginning of a set of tabular values

(B) Extrapolation near the end of a set of tabular values

(C) Interpolation near the end of a set of tabular values

(D) Interpolation near the beginning of a set of tabular values

3. Let $I = \int_0^2 \left(e^{x^2} + \frac{1}{2}\sin x^2\right)dx$.

Then :

(A) $I > 2\ (e^4 + 1)$

(B) $I < 1$

(C) $I = 2e^{\theta^2} + \sin\ \theta^2$

for some θ, $0 \leq \theta \leq 2$

(D) $I = 2\left(e^4 + \frac{1}{2}\right)$.

Or

The general first order differential equation $\frac{dy}{dx} = f(x, y)$; with initial condition $y(x_0) = y_0$,
then the nth approximation by Picard's method is :

(A) $y^{(n)} = x_0 + \int_{y_0}^{y} f(x, y^{(n-1)})dy$,

(B) $y^{(n)} = x_1 + \int_{y_0}^{y} f(x, y^{(n-1)})dy$

(C) $y^{(n)} = y_0 + \int_{x_0}^{x} f(x, y^{(n-1)})dx$

(D) $y^{(n)} = y_1 + \int_{x_0}^{x} f(x, y^{(n-1)})dx$

4. The derivative of the function $f : \mathbf{R}^2 \to \mathbf{R}^2$ given by $f(x, y) = (x + y \sin x, y + x \cos x)$ at $(0, 0)$ is the linear transformation whose matrix w.r.t. standard basis is :

(A) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Or

From the following table, the area bounded by the curve and the x-axis from $x = 7.47$ to $x = 7.52$, with $h = 0.01$, by using trapezoidal rule is :

x	$f(x)$
7.47	1.93
7.48	1.95
7.49	1.98
7.50	2.01
7.51	2.03
7.52	2.06

(A) 0.0996

(B) 0.9960

(C) 0.0556

(D) 0.5560

5. Consider the following linear programming problem :

Max Z = $x_1 + x_2$

Subject to the constraints :

$3x_1 + 2x_2 \leq 12$

$x_2 \leq 2$

x_1, x_2 are non-negative integers.

(I) $x_1 = 2, x_2 = 2$ is an optimal solution

(II) $x_1 = 3, x_2 = 1$ is an optimal solution

(III) $x_1 = 4, x_2 = 0$ is an optimal solution

(A) Only (I) is true

(B) Only (II) is true

(C) Only (II) and (III) are true

(D) All the three are true

6. There exists a bounded optimum solution to a linear programming problem, if and only if there exists a feasible solution to :

(A) Both primal and its dual

(B) only primal

(C) only dual

(D) Either of primal or dual

7. The optimal solution to the following problem is :

Max $Z = 3x_1 + 2x_2$

Subject to constraints :

$x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

and $x_1, x_2 \geq 0$

(A) $x_1 = 1$ and $x_2 = 3$

(B) $x_1 = 2$ and $x_2 = 2$

(C) $x_1 = 0$ and $x_2 = 4$

(D) $x_1 = 3$ and $x_2 = 1$

8. The optimal solution to the following problem is

Max $Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

Subject to constraints :

$x_1 + x_2 \leq 2$

and $x_1, x_2 \geq 0$

(A) $x_1 = \frac{1}{3}$ and $x_2 = \frac{5}{6}$

(B) $x_1 = \frac{5}{6}$ and $x_2 = \frac{1}{3}$

(C) $x_1 = \frac{5}{3}$ and $x_2 = \frac{1}{3}$

(D) $x_1 = \frac{1}{3}$ and $x_2 = \frac{5}{3}$

9. Which of the following is not *true* ?

(A) Let $f(z) = u(x, y) + iv(x, y)$ and u and v both are harmonic functions, then $f(z)$ is analytic

(B) If $f(z) = u + iv$ is analytic, then $u(x, y)$ and $v(x, y)$ are both harmonic functions

(C) If $u(x, y)$ and $v(x, y)$ are harmonic conjugates, then $f(z) = u + iv$ is analytic.

(D) If harmonic functions satisfy Cauchy-Riemann equations, then the function $f(z) = u + iv$ is analytic.

Or

Analysis of covariance is :

(A) a statistical technique that can be used to help equate groups on specific variables

(B) a statistical technique that can be used to control sequencing effects

(C) a statistical technique that substitutes for random assignment to groups

(D) a statistical technique that adjusts scores on the independent variable to control for extraneous variables

10. Let the integral $\int_0^{1+i} z\,dz$ be evaluated in a region $|z| = 2$ along different paths C, C_1 and C_2 between 0 and $1 + i$, where C is along the line segment OP, C_1 is along OMP and C_2 along ONP, where O = (0, 0), M = (1, 0), P = (1, 1) and N = (0, 1). Then :

(A) $\int_C z\,dz = \int_{C_1} z\,dz = \int_{C_2} z\,dz$

(B) $\int_C z\,dz = \int_{C_1} z\,dz \neq \int_{C_2} z\,dz$

(C) $\int_{C_1} z\,dz = \int_{C_2} z\,dz \neq \int_C z\,dz$

(D) $\int_C z\,dz \neq \int_{C_1} z\,dz \neq \int_{C_2} z\,dz$

Or

Which of the following correlations is the strongest ?

(A) –1.00

(B) +0.90

(C) +0.10

(D) –0.95

11. Let $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} \middle| x \in \mathbf{R} \right\}$.

Consider the following statements :

(1) (M, •) is not a group

(2) (M, •) is a group

(3) (M, +, •) is a commutative ring with unity

(4) (m, +, •) is a field

Then :

(A) only (1) is true

(B) only (2) and (3) are true

(C) only (3) and (4) are true

(D) Each of (2), (3) and (4) is true

Or

Last year, a small statistical consulting company paid each of its five statistical assistants Rs. 2,20,000, two statistical analysts Rs. 5,00,000 each, and the senior statistician Rs. 27,00,000. The number of employees earning less than the average salary is :

(A) 4

(B) 5

(C) 6

(D) 7

12. Let G and H be finite cyclic groups. Then G $\oplus$ H is cyclic iff :

(A) O(G) and O(H) are relatively prime

(B) Both O(G) and O(H) are composite numbers

(C) Exactly one of O(G) and O(H) is prime

(D) O(G) = O(H)

Or

In measuring the centre of the data from a skewed distribution, the median would be preferred over the mean for most purposes because :

(A) the mean may be heavily influenced by the larger observations and this gives too high an indication of the centre

(B) the median is smaller than the mean and smaller numbers are always appropriate for the centre

(C) the mean measures the spread in the data

(D) the median measures the arithmetic average of the data excluding outliers.

13. The splitting field of $x^4 - a$ over Q is :

(A) $\mathbf{Z}_4$

(B) $\mathbf{Q}$

(C) $\mathbf{R}$

(D) $\mathbf{Q}\left(\sqrt[4]{a},\ w\right)$

where w is a primitive fourth root of unity.

Or

Consider the Borel space (R, **B**). Define $F(x) = \left(1 - e^{-x}\right) I_{[x \geq 0]}$ and

$$G(x) = I_{[x \geq 0]}$$

where I_A is the indicator function of the set A. Let λ_F and λ_G be the Lebesgue-Stieltje's measure generated by F and G respectively. Which of the following statements are *correct* ?

(*i*) λ_F is dominated by λ_G

(*ii*) λ_G is dominated by λ_F

(*iii*) μ is a probability measure in R, if for any $B \in \mathbf{B}$, $\mu(B) = \frac{1}{2}\lambda_F(B) + \frac{1}{2}\lambda_G(B)$

(*iv*) If $X(w) = e^{w/2}$, then X is a random variable on (R, **B**, μ) and $E(X) = \frac{3}{2}$

(A) (*i*) and (*iii*)

(B) (*ii*) and (*iv*)

(C) (*i*) and (*iv*)

(D) (*iii*) and (*iv*)

14. Let $f : \mathbf{R}^3 \to \mathbf{R}^3$ be defined by $f(x, y, z) = (2x + y - z , x - y, x + z)$, where the basis of the domain is the usual basis and that of the codomain is $\{(1, 1, 1), (1, 1, 0) (1, 0, 0)\}$.

Then the matrix of f relative to these bases is :

(A) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 2 & -1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Or

Let Ω be a non-empty set. Let $v(A)$ = number of elements of the set $A \subset \Omega$ and $\mathbf{F} = \{A \subset \Omega :$ either $v(A)$ is finite or $v(A^c)$ is finite$\}$

Which of the following are *correct* ?

(*i*) ϕ and $\Omega \notin \mathbf{F}$

(*ii*) $\mathbf{F}$ is a field

(*iii*) $\mathbf{F}$ is a σ – field, if $v(\Omega) = \infty$

(*iv*) Let $v(\Omega) = \infty$ and $\mathbf{F_1} = \{A \subset \Omega$; A is countable or A^c is countable$\}$.

Then $\mathbf{F}_1$ = minimal σ–field generated by $\mathbf{F}$.

(A) (*i*) and (*ii*)

(B) (*ii*) and (*iii*)

(C) (*ii*) and (*iv*)

(D) (*iii*) and (*iv*)

15. The ring Z[i] is :
(A) a Euclidean domain
(B) not a unique factorization domain
(C) a PID but not a Euclidean domain
(D) neither a PID, nor a Euclidean domain

Or

Which of the following statements are true about a characteristic function $\phi_X(t) = E\left(e^{itX}\right)$ of a random variable X ?
(i) For all $t \in R$, $|\phi_x^{(t)}| \geq 1$
(ii) $|\phi_X(t+h) - \phi_X(t)| \to 0$ as $h \to 0$ uniformly for all $t \in R$
(iii) For each $\alpha > 2$, $\phi_X(t) = e^{-|t|^{\alpha}}$ is not a characteristic function
(iv) The characteristic function of the random variable X with pdf $f(x) = \frac{1}{2}e^{-|x|}, x \in R$ is $\phi_X(t) = \frac{1}{\Pi\ (1 - t^2)}$.

Which of the following statements are *correct* ?
(A) (ii) and (iii)
(B) (i), (ii) and (iii)
(C) (i), (ii) and (iv)
(D) (iii) and (iv)

16. Let X be a complete metric space. Let $\{O_k\}$ be a countable collection of dense open subsets of X. Then :
(A) $\cap O_k$ is countable
(B) $\cap O_k$ is finite
(C) $\cap O_k$ is empty
(D) $\cap O_k$ is dense in X

Or

Let $F : R \to [0, 1]$ be the distribution function of a random variable X. Which of the following statements is *true* ?
(A) F has at most finite number of discontinuity points
(B) If F_1 and F_2 are two distribution functions, then $F_1 + F_2$ is also a distribution function.
(C) $F : R \to R$ defined by

$$F(x) = \begin{cases} 0 & x \leq 0 \\ \alpha + \beta e^{-x^2/2} & x > 0 \end{cases}$$

is a distribution function for each value of (α, β)
(D) F is a non-decreasing function

17. Which of the following real valued functions defined on [0, 1] is *not* dense in C[0, 1] ?
   (A) The set of Bernstein polynomials
   (B) The set of polygonal functions with rational values at nodes $\left\{\frac{0}{1}, \frac{1}{n}, \ldots \frac{n}{n}\right\}$, $n = 2, 3, 4.......$
   (C) The set of all polynomials
   (D) The set of all polynomials of degree ≤ 10

Or

The number of adults (X) living in homes on a randomly selected city block is described by the following probability distribution :

X	Prob.
1	0.25
2	0.50
3	0.15
≥ 4	?

What is the probability that 4 or more adults reside at randomly selected home ?
   (A) 0.50
   (B) 0.25
   (C) 0.15
   (D) 0.10

18. Let I_1 = the improper Riemann integral $\int_0^\infty \frac{\sin x}{x} dx$ and I_2 = the Lebesgue integral $\int_0^\infty \frac{\sin x}{x} dx$

Then :
   (A) I_1 does not exist but I_2 exists
   (B) Both I_1 and I_2 exist
   (C) I_1 exists but I_2 does not exist
   (D) Both I_1 and I_2 do not exist

Or

Bob is a high school basketball player. He is a 70% free throw shooter. That means his probability of making a free throw is 0.70. What is the probability that Bob makes his first free throw on his fifth shot ?
   (A) 0.0024
   (B) 0.0081
   (C) 0.0057
   (D) 0.1681

19. Which of the following is *not* sufficient for existence of derivative a.e. for a function $f : [a, b] \to \mathbf{R}$
(A) f is absolutely continuous
(B) f is integrable
(C) f is of bounded variation
(D) f is an indefinite integral

Or

An insurance company has estimated the following cost probabilities for the next year on a particular model of car :

Cost (Rs.)	**Probability**
0	0.60
500	0.05
1000	0.13
2000	?

The expected cost to the insurance company is (approximately) :
(A) Rs. 155
(B) Rs. 595
(C) Rs. 645
(D) Rs. 875

20. Which of the following groups is simple ?
(A) Klein four group
(B) Icosahedral group
(C) S_3
(D) Cyclic group of order 15

Or

A random variable Y has the following probability distribution :

Y	**P(Y)**
–1	3C
0	2C
1	0.4
2	0.1

The value of constant C is :
(A) 0.05
(B) 0.10
(C) 0.15
(D) 0.20

21. For which value of n, the regular n-gon can be constructed by ruler and compass ?
(A) $n = 7$
(B) $n = 11$
(C) $n = 17$
(D) $n = 27$

Or

Molly earned a score of 940 on a national achievement test. The mean test score was 850 with a standard deviation of 100. What proportion of students had a higher score than Molly ? (Assume that test scores are normally distributed) :
(A) 0.10
(B) 0.50
(C) 0.18
(D) 0.82

22. Let $\alpha = \sqrt[3]{2}$, $\xi = \frac{1}{2}\left(-1 + \sqrt{-3}\right)$, $\beta = \alpha\,\xi$

Then the degree of $\alpha + \beta$ over Q is :

(A) Three

(B) Same as the degree of $\alpha - \beta$

(C) Six

(D) Five

Or

A random sample of 5 people is taken from a population in which 40% favour a particular political stand. What is the probability that exactly 3 individuals in the sample favour this political stand ?

(A) 0.2304

(B) 0.1332

(C) 0.3118

(D) 0.0164

23. Let p be a prime number. Then which of the following statements is not *true* ?

(A) There is an irreducible polynomial of degree n over $\mathbf{Z}_p$ for some integer $n > 2$

(B) There is no irreducible polynomial of degree 101 over $\mathbf{Z}_p$

(C) There is an irreducible polynomial of degree n over $\mathbf{Z}_p$ for every positive integer n

(D) If $[F : \mathbf{Z}_p] = n$, for some positive integer n, then F is a splitting field of a polynomial of degree n over $\mathbf{Z}_p$

Or

Which of the following is *not* an assumption of binomial distribution ?

(A) All trials must be identical

(B) The probability of success in the trials is equal to 0.5

(C) All trials must be independent

(D) Each trial must be classified as a success or a failure

24. The kernel of a bounded linear operator on a normed linear space need not be :

(A) convex

(B) linear

(C) open

(D) closed

Or

The probability that a certain machine will produce a defective item is $\frac{1}{4}$. If a random sample of 6 items is taken from the output of this machine, what is the probability that there will be 5 or more defectives in the sample ?

(A) $\frac{1}{4096}$

(B) $\frac{4}{4096}$

(C) $\frac{19}{4096}$

(D) $\frac{18}{4096}$

25. Let X be a normed linear space and $x, y \in X$ such that $x \neq y$. Then :

(A) there is no bounded linear functional f such that $f(x) = f(y)$

(B) there are no bounded linear functionals f and g such that $f(x) = \|x\|$ and $g(y) = \|y\|$

(C) there is a bounded linear functional f such that $f(x) \neq f(y)$

(D) for every non-zero bounded linear functional f we have $f(x) \neq f(y)$

Or

It is supposed that eating pre-sweetened cereal tends to increase the number of dental cavities in children. A sample of children was entered into a study (with parental consent) and followed for several years. Each child was classified as loving sweetened cereal or not loving sweetened cereal. At the end of the study, the extent of tooth damage was measured. The summary data is as follows :

Group	n	Mean	Std. dev.
Sweetened	10	6.41	5.0
Non-sweetened	15	5.20	15.0

An approximate 95% confidence interval for the difference in the mean tooth damage is :

(A) $(6.41 - 5.20) \pm 2.26 \sqrt{\frac{5}{10} + \frac{15}{15}}$

(B) $(6.41 - 5.20) \pm 1.96 \sqrt{\frac{25}{10} + \frac{225}{15}}$

(C) $(6.41 - 5.20) \pm 2.26 \sqrt{\frac{25}{10} + \frac{225}{15}}$

(D) $(6.41 - 5.20) \pm 2.07 \sqrt{\frac{146}{10} + \frac{146}{15}}$

26. Let F be a one-one onto continuous linear transformation of a Banach space X onto a Banach space Y. Then the inverse $F^{-1} : Y \to X$

(A) may not be continuous

(B) is linear and continuous

(C) may not be linear

(D) may not be one-one and onto

Or

The following statement can be made about non-parametric tests :

(A) They require large samples

(B) They can be applied to ordinal data

(C) Student's t-test is a non-parametric test

(D) They cannot be used to analyse samples that are normally distributed

27. Let f be a bounded linear functional on a Hilbert space H. Then :

(A) there is unique $y \in$ H such that $\|f\| = \|y\|$ and $f(x) = (x, y)$ for all $x \in$ H.

(B) there is $y \in$ H such that $f(x) = (x, y)$ for all $x \in$ H; however such a y need not be unique.

(C) for any $y \in$ H with $\|f\| = \|y\|$, $f(x) = (x, y)$ all for $x \in$ H

(D) for $y \in$ H with $f(x) = (x, y)$ for all $x \in$ H, $\|f\| \neq \|y\|$

Or

Non-parametric tests include :

(A) Chi-squared test

(B) Student's t-test

(C) Snedecor's F-test

(D) ANOVA

28. A bijective continuous function $f : X \to Y$ is a homeomorphism if :

(A) X and Y are Hausdorff

(B) X is Hausdorff and Y is compact

(C) X is compact and Y is Hausdorff

(D) X is compact and Y is connected

Or

Paired t-test :

(A) is used only on large number of patients

(B) is not sutiable for samples smaller than 20

(C) applies to normal distribution

(D) is used for two independent samples

29. Let Y be a subspace of a topological space X. Which of the following statements is *not true* ?

(A) If X is first countable, then so is Y

(B) If X is second countable, then so is Y

(C) If X is Hausdorff, then so is Y

(D) If X is separable, then so is Y

Or

Suppose X has Bernoulli distribution with success probability p. If prior distribution of p is $v(0, 1)$, then Bayes estimator of p under squared error loss function based on a single observation x is :

(A) $\frac{x+1}{2}$

(B) $\frac{x+2}{3}$

(C) $\frac{x+2}{2}$

(D) $\frac{x+1}{3}$

30. Let X be a connected normal space having more than one point. Then :

(A) X is finite

(B) X is countable infinite

(C) X is uncountable

(D) X is countable

Or

Let, $x_1, x_2, \ldots x_n$ be a random sample from $f_\theta(x) = \frac{1}{\theta}$, $k\theta \le x \le (k+1)\theta$,

$k > 0$ and known

$= 0$ otherwise.

MLE of θ is :

(A) $\frac{x_{(1)}}{k}$

(B) $x_{(n)}$

(C) $\frac{x_{(n)}}{k+1}$

(D) $\frac{x_{(1)} + x_{(n)}}{2}$

31. The fundamental group of the circle is :

(A) trivial

(B) non-abelian

(C) infinite cyclic

(D) finite cyclic

Or

A sample of size 1 is taken from an exponential distribution with mean θ. To test $H_0 : \theta = 1$ against $H_1 : \theta > 1$ the test rejects H_0 if $x > 2$. The size of the test is :

(A) $1 - e^{-2}$

(B)

(C) e^{-2}

(D) $1 - \frac{1}{2}e^{-2}$

32. The number of positive integers less than or equal to 100 and not divisible by any of 2, 3, and 5 is :

(A) 26

(B) 50

(C) 33

(D) 20

Or

Let $\delta(x)$ be UMVUE of θ. Let B = CRLB for the variance of an unbiased estimator of θ, then :

(A) $v(\delta(x)) \geq \text{B}$

(B)

(C)

(D)

33. Let L be a modular lattice. Then L is Boolean if :

(A) L does not contain M_3 as a sublattice

$v(\delta(x)) \not\lessgtr \text{B}$

(B) L is uniquely complemented

(C) L is complemented

(D) L is pseudocomplemented

Or

If the Binomial frequencies 35 and 9 are expected to occur in the ratio 3 : 1, the value of Chi-square statistic is approximately :

(A) 5.10

(B) 0.48

(C) 1.45

(D) 2.40

34. Which of the following is a planar graph ?

(A) K_{10}

(B) $K_{5,3}$

(C) Any graph with four vertices and twelve edges

(D) Any graph in which every vertex is either a pendant or a cut vertex

Or

Significance of multiple correlation coefficient is tested using :

(A) z test

(B) T-test

(C) F-test

(D) Chi-square test

35. Let P be the poset of all subgroups of a cyclic group on ten elements, with inclusion as a partial order. Then :

(A) P is a non-modular lattice

(B) P is not a lattice

(C) P is a distributive lattice

(D) P is a chain

Or

The degrees of freedom associated with 3×4 contingency table for testing independence are :

(A) 12

(B) 8

(C) 6

(D) 9

36. Consider the following *two* statements :

(I) If f is a real valued function defined on a rectangle R such that exists, continuous on R and $\left|\frac{\partial f}{\partial y}\right| \leq k$ $(x, y) \in$ R for some $k > 0$.

(II) f satisfies the Lipschitz condition on R

Then :

(A) (I) $\Rightarrow$ (II)

(B) (II) $\Rightarrow$ (I)

(C) (I) $\Leftrightarrow$ (II)

(D) (I) $\not\Leftrightarrow$ (II)

Or

Let $x_{(n)}$ denote the nth ordered statistic from a random sample of size n from $v(0, \theta)$. Then $E[X_{(n)}]$ is given by :

(A)

(B) $\frac{n + 1}{n}\theta$

(C) θ

(D) $\frac{n - 1}{n}\theta$

37. The characteristic curves of the second order partial differential equation :

$x^2u_{xx} - y^2u_{yy} = 0$

are :

(A) $x + y = c_1, x - y = c_2$

(B) $x^2 + y^2 = c_1, x^2 - y^2 = c_2$

(C) $x^2 + xy + y^2 = c_1, x^2 - xy + y^2 = c_2$

(D) $xy = c_1, \frac{y}{x} = c_2$

Or

If the correlation matrix ρ is :

$$\rho = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}$$

then $\rho_{12.3}$ is :

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{1}{3}$

38. The partial differential equation $y^2 z_{xx} - z_{yy} + 4z = 0$, $y^2 < 0$ represents :

(A) elliptic equation

(B) parabolic equation

(C) hyperbolic equation

(D) wave equation

Or

(X, Y) has bivariate normal distribution with $\mu_x = 1$, $\mu_y = 2$, $\sigma_x^2 = 2$, $\sigma_y^2 = 1$, cov(X, Y) = 0.5. If X = 0, then the conditional (mean, variance) of Y is given by :

(A) $\left(\frac{7}{4}, \frac{7}{8}\right)$

(B) $\left(\frac{15}{4}, \frac{7}{16}\right)$

(C) (2, 1)

(D) $\left(\frac{15}{8}, \frac{1}{8}\right)$

39. The hypergeometric form of the Binomial expansion $(1 - x)^{-n}$ is given by :

(A) $2F_1\ (n, b, b; x)$

(B) $2F_1\ (-n, 1, -; x)$

(C) $2F_1\ (n, -b, b; x)$

(D) $2F_1\ (-n, -, 1; x)$

Or

If (X, Y) follow bivariate Normal distribution with $\mu_X = \mu_Y = 0$, $\sigma_X^2 = \sigma_Y^2 = 1$ and correlation coefficient $\rho_{X,Y} = \frac{1}{2}$, which of the following *does not* have Chi-square distribution ?

(A) $X^2 + Y^2$

(B) X^2

(C) Y^2

(D) $\frac{4}{3}(X^2 - XY + Y^2)$

40. The number of primitive roots of 53 is :

(A) zero

(B) 1

(C) 20

(D) 24

Or

If X follows $N_P(\mu, \Sigma)$, where

$$\Sigma = \begin{bmatrix} 1 & .4 & .4 \\ .4 & 1.0 & .4 \\ .4 & .4 & 1.0 \end{bmatrix}$$

then the test statistic for testing $H_0 : \mu = \mu_0$ is distributed as :

(A) Normal

(B) Student's t

(C) F

(D) Chi-square

41. Let $\tau(x)$ denote the number of positive divisors of the positive integer x.

If $\tau(x) = 6$, then x is :

(A) 6

(B) 15

(C) 12

(D) 7

Or

For children between the ages of 18 months and 29 months, there is approximately a linear relationship between "height" and "age". The relationship can be represented by $y = 64.93 + 0.63(X)$, where Y represents height (in centimeters) and X represents age (in months). Joseph is 22.5 months old and is 80 centimeters tall. What is Joseph's residual ?

(A) 79.1 cms

(B) 6.9 cms

(C) 0.9 cms

(D) 56.6 cms

42. Consider the following congruences :

(1) $x \equiv 3 \pmod 4$

(2) $x \equiv 11 \pmod 8$

(3) $x \equiv 13 \pmod{16}$

Then :

(A) (1) and (2) have a common solution but (1) and (3) do not have a common solution

(B) All the three congruences have a common solution

(C) (1) and (3) have a common solution but (2) and (3) do not have a common solution

(D) Any two of the congruences have a common solution

Or

A study was conducted to examine the quality of fish after seven days in ice storage. For this study :

Y = measurement of fish quality (on a 10 point scale with 10 = BEST)

X = # of hours after being caught that the fish were packed in ice

The simple linear regression line is : Y = 8.5 – 0.5X. From this we can say that :

(A) A one hour delay in packing the fish in ice decreases the estimated quality by 0.5

(B) A one hour delay in packing the fish in ice increases the estimated quality by 0.5

(C) If the estimated quality increases by 1, then the fish have been packed in ice one hour sooner

(D) If the estimated quality increases by 1, then the fish have been packed in ice one hour later

43. The number of positive integers less than or equal to 345 and relatively prime to 345 is :

(A) 100

(B) 168

(C) 176

(D) 173

Or

Which of the following is *not true* ?

(A) In some experiments, different samples contain different numbers of observations. However, the concepts and methods of single factor ANOVA are most easily developed for the case of equal sample size

(B) The population distributions in single-factor ANOVA are all assumed to be normally distributed with the same variance

(C) In one-way ANOVA, the population distributions are all not assumed to be normally distributed with the same variance

(D) In one-way ANOVA, F test is used to test the equality of treatment means

44. If $T = \frac{1}{2}m\dot{r}^2$, $V = \frac{1}{r}\left(1 + \frac{\dot{r}^2}{c^2}\right)$ are kinetic energy and potential energy respectively of a particle, then :

(A) the Hamiltonian of the particle is conserved and represents total energy

(B) Hamiltonian of the particle represents energy

(C) Hamiltonian of the particle is conserved

(D) Neither Hamiltonian of the particle represents total energy nor it is conserved

Or

In single-factor ANOVA, MSE is the mean square for error, and MST*r* is the mean square for treatments. Which of the following statements is *not true* ?

(A) The value of MST*r* is affected by the status of (true or false)

(B) MST*r* is a measure of between-samples variation

(C) The value of MSE is affected by the status of (true or false)

(D) MSE is a measure of within-samples variation

45. The potential energy of a particle of mass m moving on a cycloid $S = 4a \sin \theta$ under gravity is given by :

(A) $V = mg\, a\, (1 - \cos 2\theta)$

(B) $V = mg\, a\, (\theta - \cos \theta)$

(C) $V = mg\, a\, (2\theta + \sin 2\theta)$

(D) $V = 4\, mg\, a \sin \theta$

Or

Stratified sampling is more efficient than simple random sampling because :

(A) Strata are homogeneous leading to minimal internal variation

(B) Strata partition the population into mutually exclusive and exhaustive subpopulations

(C) Sample selection from all strata ensures representation of the entire population in the sample

(D) Strata are small and therefore sampling variation is also small

46. The number of degrees of freedom of a particle of mass m thrown up an inclined plane which is moving with constant velocity v is :

(A) 2

(B) 1

(C) 3

(D) 4

Or

Cluster sampling is better than simple random sampling if :

(A) Clusters are formed by selecting consecutive sampling units from a sorted population

(B) Clusters are formed so as to miximize within-cluster variation

(C) Clusters are formed so as to have equal variances

(D) clusters are formed with help of random number tables

47. If a rigid body with one point fixed rotates about the principal axis of the body, then during motion of the body :

(A) only kinetic energy of the body is conserved

(B) angular momentum of the body is conserved

(C) the magnitude of the angular momentum of the body is conserved

(D) both kinetic energy and magnitude of the angular momentum of the body are conserved

Or

Ratio estimator is more efficient than the sample mean if :

(A) the concomitant variable is independent of the variable of interest

(B) the sample ratio is unbiased for the population ratio

(C) the concomitant variable has smaller variance than the variable of interest

(D) the variable of interest is approximately proportional to the concomitant variable

48. The linearly independent tangent vector field(s) to the surface M : $z = xy$ is (are) :

(1) $(-y, -x, 1)$

(2) $(x, 0, z)$

(3) $(x, -y, 0)$

(4) $(0, y, z)$

then :

(A) Only (1) is true

(B) Only (1), (2) are true

(C) Only (1), (3) are true

(D) Only (2), (3), (4) are true

Or

The sampling variance of the Horvitz-Thompson estimator can be negative because :

(A) it assumes unequal probabilities of selection for sampling units

(B) some individual values can exceed the average

(C) larger values are assigned higher probabilities of selection

(D) individual values can be negative even if the expected values are positive

49. The profile curve of the torus of revolution :

M : $\left(\sqrt{x^2 + y^2} - 4\right)^2 + z^2 = 4$

about z-axis :

(A) $(x - 4)^2 + z^2 = 4$

(B) $(y - 4)^2 + z^2 = 4$

(C) $x^2 + z^2 = 4$

(D) $y^2 + z^2 = 4$

Or

If Y is a vector of observations, T is the vector of treatment totals, B is a vector of block totals and I is a vector whose every element is unity, consider the following :

(1) I'Y

(2) I'T

(3) I'B

then which of the following quantities represents the grand total ?

(A) All (1), (2), (3)

(B) Only (2), (3)

(C) Only (1), (3)

(D) Only (1), (2)

50. Geodesics on a surface with metric :

$ds^2 = dr^2 + r^2\, d\theta^2$

are :

(A) helixes

(B) great circles

(C) straight lines

(D) parabolas

Or

For a BIBD with parameters $v = b = 7$, $r = k = 3$ and $\lambda = 1$, the number of treatments common in the first and last block is :

(A) 2

(B) 3

(C) 1

(D) 4

51. Consider the following *two* statements :

(I) A surface is minimal iff asymptotic lines are orthogonal

(II) A surface is minimal if the mean curvature is zero at all points of the surface

Then

(A) both (I) and (II) are not true

(B) both (I) and (II) are true

(C) only (I) is true

(D) only (II) is true

Or

The highest order of interaction in a 2^3-factorial experiment is :

(A) I

(B) II

(C) III

(D) IV

52. The extremal of the functional

$$I = \int_{x_1}^{x_2} x\sqrt{1 + y'^2}\, dx$$

is a :

(A) catenary

(B) cycloid

(C) arc of the great circle

(D) circle

Or

If X denotes the information matrix of design, then D-optimality criterion deals with :

(A) Minimization of $|(X'X)^{-1}|$

(B) Minimization of trace $(X'X)^{-1}$

(C) Maximization of $|(X'X)^{-1}|$

(D) Maximization of trace $(X'X)^{-1}$

53. The extremizing functional :

$$I(y(x)) = \int_0^{2\pi} \left(y'^2 - y^2\right)dx,\ y(0) = 1,$$

$y(2\pi) = 1$

has :

(A) a unique solution

(B) exactly two solutions

(C) an infinite number of solutions

(D) no solution

Or

Consider a time series model $X_t = \sigma_t \varepsilon_t$ with $\sigma_t^2 = \alpha_0 + \alpha_1 X_{t-1}^2$, $\varepsilon_t \sim$ iid Normal (0, 1). Which of the following statements are true for this model ?

(*i*) $E(X_t) = 0$, $Var(X_t) = \dfrac{\alpha_0}{(1 - \alpha_1)}$, $\alpha_1 < 1$

(*ii*) $\{X_t^2\}$ has a MA(1) representation

(*iii*) $E(X_t^2 | X_{t-1}, X_{t-2}, ...) = \sigma_t^2/(1 - \alpha_1)$

(*iv*) $E(X_t X_{t+h}) = 0$ for $h > 0$

(A) (*i*) and (*iii*)

(B) (*ii*) and (*iv*)

(C) (*i*) and (*iv*)

(D) (*ii*) and (*iii*)

54. The first integral of the Euler-Lagrange's equation of the functional :

$$\int_a^b \left(\frac{1}{2} m\, \dot{y}^2 - cy^2\right) dx,$$ where a, b, c are constants and $\dot{y} = \frac{dy}{dx}$, is :

(A) $m\dot{y}^2 + 2cy^2 = k$
(B) $m\ddot{y} + 2cy = 0$
(C) $m\dot{y} + 2cy = k$
(D) $m\dot{y} - 2cy^2 = k$
where k is a constant

Or

Let $Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + Z_t$, $Z_t \sim$ white noise $(0, \sigma^2)$ be an autoregressive process of order two. Suppose we have the following conditions on the parameters of the process $\{Y_t\}$:

(*i*) $\phi_1 + \phi_2 < 1$
(*ii*) $\phi_1 - \phi_2 > -1$
(*iii*) $-1 < \phi_2 < +1$
(*iv*) $-1 < \phi_1 < +1$

Which of the above conditions leads $\{Y_t\}$ to be a causal process ?

(A) (*i*), (*ii*) and (*iii*)
(B) (*ii*) and (*iv*)
(C) (*i*), (*ii*) and (*iv*)
(D) (*i*) and (*ii*)

55. The minimal of the functional :

$$I(y(x)) = \int_0^1 \left(\frac{y'^2}{2} + yy' + y' + y\right) dx$$

occurs on the curve :

(A) $y = \frac{x^2}{2} - 2x + 3$
(B) $y = \frac{x^2}{2} + 3x + 1$
(C) $y = \frac{x^2}{2} - 3x + 1$
(D) $y = \frac{1}{2}\left(x^2 - 3x + 1\right)$

Or

For a simple one-step exponential smoother of a given time series $\{y_t\}$, which of the following statements are *correct* ?

(*i*) $S_t = \alpha\, y_t + (1 - \alpha)\, S_{t-1}$ with $0 < \alpha < 1$ and $S_1 = y_1$
(*ii*) Large value of α is chosen for the forecast to be responsive to recent observations.
(*iii*) If y_t's are uncorrelated, $E(y_t) = \mu$, and $V(y_t) = \sigma^2$, then $E(S_t - \mu)^2 > \sigma^2$
(*iv*) It remove the seasonality of the series effectively.

(A) (*ii*) and (*iii*)
(B) (*i*) and (*iv*)
(C) (*i*) and (*iii*)
(D) (*i*) and (*ii*)

56. The integral equation :

$$x(t) = f(t) + \frac{1}{\pi}\int_0^{2\pi} \sin(t+s)x(s)ds$$

possesses infinitely many solutions for :

(A) $f(t) = 1 - t$

(B)

(C)

(D)

Or

Consider an AR(2) process $X_t(t) = X1_{t-1} + t$ $-\frac{1}{2}X_{t-2} + Z_t$, where $Z_t \sim$ white noise $(0, \sigma^2)$. Which of the following statements are *correct* about this process ?

(*i*) the process is stationary

(*ii*) the process is invertible

(*iii*) the first order autocorrelation $\rho(1) = \frac{1}{3}$

(*iv*) the process has infinite variance

(A) (*i*) and (*iv*)

(B) (*i*) and (*iii*)

(C) (*i*) and (*ii*)

(D) (*ii*) and (*iii*)

57. The eigen values of the integral equation :

$$x(t) = \lambda\int_0^1 \left(s\sqrt{t} - t\sqrt{s}\right)ds$$

are :

(A) $1 + i\sqrt{150}, 1 - i\sqrt{50}$

(B) $1 + \sqrt{150},\ 1 - \sqrt{150}$

(C) $i\sqrt{150}, -i\sqrt{150}$

(D) $\sqrt{150},\ -\sqrt{150}$

Or

A stochastic process $\{X(t), t \geq 0\}$ with independent increments is said to have stationary increments if :

(A) $X(t)$ and $X(t+h)$ are independent

(B) $X(t) - X(s)$ and $X(t+h) - X(s+h)$ for all $s, t, h \geq 0$ $s < t$, are independent random variables

(C) $X(t) - X(s)$ and $X(t+h) - X(s+h)$ for all $s, t, h \geq 0, s < t$, have same distribution

(D) $X(nt) - X(n(t-1))$ for all $t \geq 0$, $n = 1, 2, 3....$have same distribution

58. If $x(t)$ is continuous and satisfies

$$x(t) = \int_0^t (1 - t)s\, x(s)ds + \int_t^1 (1 - s)t\, x(s)ds,$$

then $x(t)$ is also the solution of :

(A) $x''(t) - 2t\, x(t) = 0$, $x(0) = x'(1) = 1$

(B) , $x(0) = x'(1) = 1$

(C) , $x(0) = x(1) = 0$

(D) $x''(t) + x(t) = 0$, $x(0) = x(1) = 0$

Or

A cell phone repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repairs cell phones in the order in which they come in, and if the arrival of sets is approximately Poisson with an average rate of 10 per eight hours day. Then the expected idle time of the repairman is :

(A) 5 hours

(B) 4 hours

(C) 2 hours

(D) 3 hours

59. Let $x(t) = f(t) + \lambda$,

where $f(t)$ and $k(t, s)$ are known functions, a and b are constants and λ is a parameter. If λ_i be the eigen values of the corresponding homogeneous equation, then the integral equation has ϕ_n general :

(A) either many solutions or no solution at all for $\lambda = \lambda_i$ depending on the form of $f(t)$,

(B) many solutions for $\lambda \neq \lambda_i$

(C) no solution for $\lambda \neq \lambda_i$

(D) a unique solution for $\lambda = \lambda_i$

Or

If a matrix of transition probability is of order $n \times n$, then the number of equilibrium equations would be :

(A) $n + 1$

(B) n

(C) $n - 1$

(D) $n - 2$

60. The value of $\Delta\left(\frac{2^x}{x!}\right)$ (taking interval of differencing as unity) is ....

(A) $\frac{2^{(1-x)}(x-1)}{(x+1)!}$

(B) $\frac{2^x(1+x)}{(1-x)!}$

(C) $\frac{2^x(1-x)}{(1+x)!}$

(D) $\frac{2^{(1+x)}(1-x)}{(1+x)!}$

Or

The transition probability matrix of a Markov chain with four states as 1, 2, 3 and 4 is :

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then which of the following is *true* ?

(A) chain is irreducible

(B) (1, 3, 4) are communicating states

(C) (1, 2, 3) are communicating states

(D) (2, 3, 4) is a closed set

61. Given the two points $[a, f(a)]$ and $[b, (f(b)]$, the linear Lagrange polynomial that passes through these *two* points is given by :

(A) $\frac{x-b}{a-b}f(a)+\frac{x-a}{a-b}f(b)$

(B) $\frac{x-b}{a-b}f(a)+\frac{x-a}{b-a}f(b)$

(C) $\frac{x-b}{b-a}f(a)+\frac{x-a}{b-a}f(b)$

(D) $\frac{x-b}{b-a}f(a)+\frac{x-a}{a-b}f(b)$

Or

In the context of vital statistics the rate of a vital event is :

(A) The ratio of the total number of persons exposed to the risk of occurrences of that event to the total number of occurrences of the event

(B) The ratio of the total number of occurrence of the event to the total number of persons exposed to the risk of occurrence of that event

(C) The ratio of the mean of total number of occurrences of the event to the total number of persons exposed to the risk of occurrence of that event

(D) The ratio of the mean of total number of persons exposed to the risk of occurrence of that event to the total number of occurrences of the event

62. A solid of revolution is formed by rotating about the x-axis the area between the x-axis, the lines $x = 0$ and $x = 1$, a curve through the points with the following coordinates :

x	y
0.00	1.0000
0.25	0.9896
0.50	0.9589
0.75	0.9089
1.00	0.8415

The volume of the solid formed using Simpson's rule is :

(A) 2.9192

(B) 2.8192

(C) 2.7192

(D) 2.6192

Or

The principal methods used for the construction of abridged life table in the context of vital statistics :

(1) Reed-Merrell method

(2) Greville's method

(3) King's method

(4) William-Farr method

(A) (1), (2) and (4)

(B) (1), (2) and (3)

(C) (1), (3) and (4)

(D) (2), (3) and (4)

63. A cubic polynomial which takes the following values :

x	$f(x)$
0	1
1	2
2	1
3	10

by Newton's forward interpolation formula is :

(A) $4x^3 - 2x^2 + 5x + 1$

(B) $2x^3 - 7x^2 + 6x + 1$

(C) $2x^3 - x^2 + 8x + 1$

(D) $4x^3 - 3x^2 + 6x + 1$

Or

In the following life table :

Age x	L_x
90	16090
91	11490
92	8012
93	5448
94	3607
95	2320
96	1447

The ages of three men A, B and C are 90, 91 and 92 respectively. What is the probability that exactly one of them will be alive in two years time :

(A) 0.1452

(B) 0.3936

(C) 0.8548

(D) 0.6064

64. The Fourier transform of $\frac{e^{-t^2}}{2}$ is :

(A) $e^{p^{2/2}}$

(B) $e^{-p^{2/2}}$

(C) $e^{-p/2}$

(D) e^{-p^2}

Or

Given that the complete expectation of life at ages 30 and 31 for a particular group are respectively 21.39 and 20.91 years and that the number living at age 30 is 41176. What is the number that attains the age 31 ?

(A) 40170

(B) 40176

(C) 40172

(D) 40178

65. Suppose that the function $y(t)$ satisfies the differential equation : $y''(t) + 2y'(t) + y(t) = 3te^{-t}$, $t > 0$ with initial values $y(0) = 4$, $y'(0) = 2$. Then the Laplace transform of $y(t)$ is :

(A) $\frac{3}{(p+1)^4} + \frac{6}{(p+1)^2}$

(B) $\frac{6}{(p+1)^2} + \frac{4}{(p+1)}$

(C) $\frac{3}{(p+1)^3} + \frac{6}{(p+1)^2} + \frac{4}{(p+1)}$

(D) $\frac{3}{(p+1)^4} + \frac{6}{(p+1)^2} + \frac{4}{(p+1)}$

Or

A parallel system with three components having constant failure rate λ has the mean time to system failure as 25 hrs then the value of λ is :

(A) $\frac{11}{25}$

(B) $\frac{150}{11}$

(C) $\frac{25}{11}$

(D) $\frac{11}{150}$

66. The inverse Laplace transform of :

$f(p) = \frac{1}{p^n}$ exists only if n is :

(A) negative integer

(B) positive integer

(C) zero

(D) negative rational

Or

If the failure rate $r(t) = t^{-1/2}$, then the corresponding pdf is :

(A) $t^{-1/2} e^{-\sqrt{t}}$

(B) $t^{-1/2} e^{-2\sqrt{t}}$

(C) $t^{1/2} e^{-2\sqrt{t}}$

(D) $t^{1/2} e^{-\sqrt{t}}$

67. Consider the following statements :

(1) Fourier transform is linear

(2) The kernel of the Fourier transform is e^{-pt}

Which of the following is *correct* ?

(A) Only (1) is true

(B) Both (1) and (2) are correct

(C) Both (1) and (2) are false

(D) Only (2) is true

Or

A distribution F(t) with failure rate function $r(t)$ is said to have an increasing failure rate average (IFRA) if :

(A) $-\frac{1}{t}\log(1 - F(t))$ increases with $t \geq 0$

(B) $\frac{1}{t}\log(1 - F(t))$ increases with $t \geq 0$

(C) $\frac{1}{t}\log(1 - F(t))$ decreases with $t \geq 0$

(D) $-\frac{1}{t}\log(1 - F(t))$ decreases with $t \geq 0$

68. Which of the following is *not true* ?

(A) $\int_C \bar{z}\, dz = 0,\ c : |z - 2| = 3$

(B) $\int_C z dz = 0,\ c : |z - 2| = 3$

(C) $\int_C \frac{1}{z} dz = 2\pi i$, C is simple closed contour enclosing 0 and lying in $\frac{1}{2} < |z| < \frac{3}{2}$

(D) $\int_C \frac{dz}{z} = 2\pi i$, C : $|z| = 1$

Or

Which of the following situations suggests a process that appears to be operating in a state of statistical control ?

(A) A control chart in which no points fall outside the control limits and no pattern is present

(B) A control chart in which no points fall outside the control limits and some pattern is present

(C) A control chart in which no points fall outside the upper control limits

(D) A control chart in which no points fall outside the lower control limits

69. Let $f(x, y)$ be the vector field given by :

$$f(x, y) = (x^2 - 2y)\overline{i} + (y + 3x^2)\overline{j}$$

Then the line integral of $f(x, y)$ along $y = x^2$ from (–1, 1) to (1, 1) is :

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) 1

(D) $\frac{-2}{3}$

Or

Variance formula of arrivals in the service time of a customer of (M/G/1) queuing model where $\rho = \frac{\lambda}{\mu}$ is :

(A) ρ^2

(B) $\rho + \lambda^2.\sigma^2$

(C) $\rho^2 + \lambda^2.\sigma^2$

(D) $\rho + \rho^2 + \lambda^2\sigma^2$

70. The value of the line integral

$$\int_C \left(x^2 - 2xy\right)dx - 5y^2dy$$

where C is the path from (0, 0) to (1, 1) along the parabola $y = x^2$ is :

(A) $-\frac{11}{6}$

(B) $-\frac{7}{6}$

(C) $\frac{7}{6}$

(D) $-\frac{13}{6}$

Or

A repair facility shared by a large number of machines has two sequential stations with respective rates one per hour and two per hour. The cumulative failure rate of all the machines is 0.5 per hour. Assuming that the system behaviour may be approximated by the two-stage tandem queue. What is the average repair time and the probability that both service stations are idle ?

(A) 3.66 hours and 0.625

(B) 2.66 hours and 0.625

(C) 3.66 hours and 0.375

(D) 2.66 hours and 0.375

71. Which one of the following is *not true* ?

(A) $\int_C \frac{1}{z} dz = 2\pi i$,
where C : $|z| = 3$

(B) $\int_C \frac{1}{z^2} dz = 0$,
where C : $|z| = 3$

(C) $\int_C \frac{dz}{z-a} = 2\pi i$,
where C : $|z - a| = 3$

(D) $\int_C \frac{dz}{z-3} = 0$

where C : $|z - 2| = 3$

Or

If orders are placed with size determined by Economic order quantity, then the re-order costs component is :

(A) Greater than the Holding cost components

(B) Either greater than or less than the Holding cost components

(C) Less than the Holding cost components

(D) Equal to the Holding cost components

72. Let S be the set of numbers in [0, 1] having decimal expansion not containing the digit 3. Then $m(S)$ is :

(A) 0

(B) $\frac{1}{3}$

(C) $\frac{9}{10}$

(D) 1

Or

In selective inventory control techniques which of the following statements are *incorrect* ?

(1) In ABC analysis, the inventory items are classified into three categories on the basis of their usage values

(2) HML classifies items according as they are high usage, medium usage or low usage

(3) VED analysis deals with classification of items on the basis of their availability

(4) XYZ analysis is based on the classification of items according to their unit cost

(A) (2), (3) and (4)

(B) (2) and (3)

(C) (1) and (4)

(D) (1), (2) and (3)

73. Consider the following statements :

(1) Every Borel set is measurable

(2) Every measurable set is a Borel set.

Then :

(A) Only (1) is true

(B) Only (2) is true

(C) Both the statements are true

(D) Both the statements are false

Or

The solution of recursive equation in dynamic programming does involve :

(1) backward computational procedure or forward computational procedure

(2) Two types of computations according as the system is continuous or discrete

(3) number of stages that provides an optimum solution or there is an indication of an unbounded solution

(4) classical methods of optimization or tabular computational scheme to achieve the optimum solution

(A) (1), (2) and (4)

(B) (1), (2) and (3)

(C) (2), (3) and (4)

(D) (1), (3) and (4)

74. Which of the following is *false* ?

(A) Let $\{f_n\}$ be a sequence of integrable functions. Then $\liminf \int f_n dx \geq \int \liminf f_n dx$

(B) Let $\{f_n\}$ be a sequence of non-negative measurable functions. Then

$\liminf \int f_n dx \geq \int \liminf f_n dx$

(C) Let $\{f_n\}$ be a sequence of measurable functions such that $|f_n| \leq g$, where g is integrable, and let $\lim f_n = f$ a.e. Then f is integrable and $\lim \int f_n dx = \int f dx$

(D) If $\{f_n\}$ is a sequence of non-negative measurable functions such that $\{f_n(x)\}$ is monotone increasing for each x, and $f = \lim f_n$, then $\int f dx = \lim \int f_n dx$

Or

Which of the following is *not* correct in dynamic programming problem ?

(A) When classical method is used in solving a dynamic programming problem, the objective may be linear or non-linear, but the constraints must be non-linear

(B) Dynamic programming deals with the time dependent decision-making problems

(C) Linear programming problem can be solved by using dynamic programming approach

(D) Optimum solution in dynamic programming problem depends on the initial solution

75. Let (X, S, μ) and (Y, J, ν) be σ-finite measure spaces. Which of the following statements about product measures is *false* ?

(A) If μ and ν are complete measures, then so is $\mu \times \nu$

(B) If $f \in L'(\mu)$ and $g \in L'(\nu)$, then $f\,g \in L'(\mu \times \nu)$

(C) If $f \in L'(\mu \times \nu)$, then the order of integration for f is interchangeable

(D) In Fubini's theorem for $f \in L'(\mu \times \nu)$, the requirement of σ-finiteness of μ an ν is essential

Or

In the study of decision-making under uncertainty which statement is *incorrect* ?

(A) Laplace criteria converts a "problem under certainty" into a "problem under uncertainty"

(B) Events are also called states of nature

(C) Opportunity loss in the pay of matrix is also called conditional pay-off

(D) The decision-maker has no idea about the possible states of nature and their probabilities

ROUGH WORK