D–GT–M–TTB

STATISTICS

Paper—II

Time Allowed : Three Hours	Maximum Marks : 200

INSTRUCTIONS

Candidates should attempt Question Nos. 1 and 5 which are compulsory, and any THREE of the remaining questions, selecting at least ONE question from each Section.

All questions carry equal marks.

Marks allotted to each part of a question are indicated against each.

Answers must be written in ENGLISH only.

Assume suitable data, if considered necessary, and indicate the same clearly.

Unless otherwise indicated, symbols and notations have their usual meanings.

Important Note

All parts/sub-parts of a question being attempted are to be answered contiguously on the answer-book. That is, where a question is being attempted, all its constituent parts/sub-parts must be answered before moving on to the next question.

Pages left blank, if any, in the answer-book(s) must be clearly struck out. Answers that follow pages left blank may not be given credit.

Section—A

(Industrial Statistics and Optimization Techniques)

1. Answer the following : 8×5=40

(a) What is the problem of duality in linear programming? For the following linear programming problem

$$\text{Maximize } f = 2X + 3Y + 5Z$$

subject to the conditions

$$10X + 9Y + Z \leq 10$$
$$X + Y + Z \geq 30$$
$$9X + 2Y - 4Z \geq 80$$
$$X \geq 0,\ Y \geq 0,\ Z \geq 0$$

write the dual problem. Further show that dual of the dual problem is the primal problem.

(b) For a two-person zero-sum game problem, explain the meaning of the following terms :

(i) Pure and mixed strategy

(ii) Saddle point

(iii) Value of the game

The following matrices are payoff matrices for a game problem :

(1) $P = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (2) $P = \begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}$

(3) $P = \begin{pmatrix} 8 & 1 \\ 3 & 7 \end{pmatrix}$

Obtain value of the game in each case.

(c) For a Markov chain with two states and transitional probability matrix $P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}$, show that the steady-state probabilities will be given by

$$\pi_1 = \frac{p_{21}}{1 - p_{11} + p_{21}}, \quad \pi_2 = \frac{p_{12}}{1 + p_{12} - p_{22}}$$

(d) Motherboard of a laptop has four components *A*, *B*, *C* and *D*. 500 number of motherboards were subjected to accelerated operational tests. They contain the following failure data :

Component	:	*A*	*B*	*C*	*D*
Number of failures	:	10	8	40	20

Determine the system reliability of the motherboard.

(e) Show that the following function represents the failure time density function :

$$f(t) = t^2 / 2 \quad \text{for} \quad 0 < t \leq 1$$
$$= \frac{1}{2}[t^2 - 3(t-1)^2] \quad \text{for} \quad 1 \leq t \leq 2$$
$$= \frac{1}{2}[t^2 - 3(t-1)^2 + 3(t-2)^2] \quad \text{for} \quad 2 \leq t \leq 3$$
$$= 0 \quad \text{otherwise}$$

Obtain expressions for the probability of failure within time t and the reliability for time t.

Compute reliability and hazard rate at time $t = 0 \cdot 5$, $t = 1$, $t = 1 \cdot 5$, $t = 2$ and $t = 2 \cdot 5$.

2. *(a)* Explain clearly the working procedure for a double sampling plan for attributes, when two points are fixed on OC curve and plan is designed so as to minimize ATI.

What are principal and supplementary OC curves? Also discuss in detail how you will draw OC, ATI, ASN and AOQ curves for this plan.

Give the working procedure for the following multiple sampling plan : 14

$n_1 = 50,\ c_1 = 2,\ n_2 = 100,\ c_2 = 3,\ n_3 = 150,\ c_3 = 8$

(b) Ten samples each of size 50 are drawn from an on-line production process and the number of defectives found is recorded as under :

8, 4, 6, 7, 3, 5, 6, 12, 8, 4

Draw the control chart for defectives and give your comments. If the process is not under control, how will you revise the chart? 12

(c) There are three retail stores S_1, S_2 and S_3, where the produced units are to be transported and their requirements are 400, 100 and 500 units respectively.

These units can be transported from three godowns G_1, G_2 and G_3, and the available units at these godowns are 200, 300 and 500 respectively.

Unit transportation cost matrix for the above is given below :

Stores / Godowns	S_1	S_2	S_3
G_1	7	3	4
G_2	2	1	3
G_3	3	4	6

Determine the optimum allocation for transporting units from godowns to retail stores, so that the total cost of transportation is minimum. 14

3. *(a)* Explain the meaning of the following terms :

(i) Lead time

(ii) Scheduling period

(iii) Order level

(iv) Buffer stock

(v) Probabilistic demand

Discuss fully order-level-lot size inventory model, stating the assumptions underlying it. Obtain expression for total inventory cost and determine the optimum values of the decision variables, so that the total inventory cost is minimum. 12

(b) A company produces two products *A* and *B* during a given time period. Each of these products has to undergo four different manufacturing operations

namely grinding, turning, assembly and testing. The manufacturing time requirements in hours per unit for these operations are given below :

Operation	*Product*	
	A	*B*
Grinding	2	4
Turning	6	2
Assembly	12	6
Testing	10	8

The average capacity for these operations in hours is 60 for grinding, 120 for turning, 400 for assembly and 400 for testing. All units produced are saleable in the market and profit contribution is ₹ 2 each for product *A* and ₹ 3 each for product *B*. Formulate linear programming problem and determine the optimum solution graphically to maximize profit. 14

(c) Derive differential-difference equation for M|M|1 : (∞|FIFO) queuing system. State the assumptions underlying it. Obtain steady-state solution for the system. Also find expressions for—

(i) average queue length;

(ii) average number of units in the system;

(iii) idle and busy period for the server. 14

4. *(a)* Explain the following :

(i) Reliability of a component

(ii) Reliability for a series system

(iii) Reliability for a parallel system

(iv) Reliability bathtub curve

Discuss briefly how reliability engineering helps in business promotional plans.

A certain system consists of 10 (ten) identical units which are connected in series. It is desired that the system reliability should be 0·95. Determine how good the component should be. 12

(b) Give lucid exposition of any *two* of the following : 12

(i) Item-by-item sequential sampling plan

(ii) Cumulative sum control charts

(iii) Renewal theory

(c) A bakery keeps stocks of popular brand bread. By past experience, the following probability distribution for daily demand is found as under :

Daily demand (units)	0	10	20	30	40	50
Probability	0·12	0·08	0·30	0·10	0·20	0·20

From the above—

(i) calculate average daily demand for bread based on the probability distribution;

(ii) use the following sequence of random numbers to simulate demand for next 10 days :

45, 26, 32, 37, 59, 88, 07, 15, 72, 04

(iii) find out the stock situation for the next 10 days if the bakery-man decides to make 40 breads per day;

(iv) estimate average daily demand based on simulated data for the next 10 days;

(v) find out the average daily stock situation based upon the simulated data for the next 10 days. 16

Section—B

(Quantitative Economics and Official Statistics)

5. Answer the following : 8×5=40

(a) Write the basic assumptions underlying k-variate general linear model

$$\underline{Y} = X\underline{\beta} + \underline{U}$$

with usual notations.

Establish the following for the above model :

(i) $\hat{\underline{\beta}} = (X'X)^{-1} X'\underline{Y}$

(ii) $V(\hat{\underline{\beta}}) = (X'X)^{-1} \sigma^2$

(iii) $\hat{\sigma}^2 = \underline{e}'\underline{e} / (n - k)$

(b) What are time and factor reversal tests of index numbers? Examine whether the following index numbers satisfy these tests or not :

(i) $I = \frac{1}{2}(L + P)$

(ii) $I = \sqrt{L \times P}$

where L is Laspeyres's index number and P is Paasche's index number. What are the names given to above *(i)* and *(ii)* index numbers?

(c) Discuss briefly the role of NSS organization in developing, collecting and implementing (analyzing and using) Indian statistical data.

(d) Explain different components of a time series giving their use and importance. Discuss how you will separate trend, seasonal and random component from a given time series data.

(e) The following data give production of sandalwood in a forest area during the years 1981 to 1987 :

Year (t)	:	1981	1982	1983	1984	1985	1986	1987
Sandalwood ('00 metric tons)	:	15	26	43	64	95	130	171

Fit the following trend model to the above data :

$$Y = \alpha + \beta t + \gamma t^2 + U$$

where α, β, γ are parameters and U is the disturbance term.

Give your comments on the fitted model. Also estimate production of sandalwood for the year 1988.

6. *(a)* What are generalized least squares estimators? Stating the basic assumptions underlying this model, obtain expression for generalized least squares estimators. Also find the variance of these estimators and compare them with Ordinary Least Squares (OLS) estimators. 14

(b) For estimating simultaneous linear system of equations, how would you use the following methods of estimation and why? 12

(i) Indirect Least Squares (ILS) method

(ii) Two-Stage Least Squares (2-SLS) method

(c) What are stationary time series? Explain their importance. Discuss the application of correlogram analysis and periodogram analysis for stationary time series. 14

7. *(a)* Explain the nature and applications of Gompertz curve and modified Gompertz curve.

Discuss in detail any one method for fitting these curves to a given population data. 14

(b) What are autocorrelated disturbances in linear models? Explain the effect of the presence of such disturbances. Describe 'D-W' test for detecting autocorrelation. Discuss Cochran-Orcutt interactive procedure to deal with the problem of autocorrelation. 14

(c) Explain the following, giving suitable illustrations : 12

(i) Fixed base and chain-based index numbers

(ii) Deflating of index numbers

(iii) Splicing of index numbers

8. (a) Explain the following, giving illustrations : 12

(i) Crude Birth Rate (CBR)

(ii) General Fertility Rate (GFR)

(iii) Age-Specific Fertility Rate (ASFR)

(iv) Total Fertility Rate (TFR)

(v) Gross Reproduction Rate (GRR)

(vi) Net Reproduction Rate (NRR)

(b) What is a life table? Explain various terms and concepts used in life tables and discuss the importance and uses of life tables. How would you construct a life table for given population data? 14

(c) Discuss how official statistics pertaining to trade, production and prices are collected in India. Give complete procedure. Also list some important publications for these official statistics with special reference to India. 14

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