VITEEE Mathematics 2013

1. If N denote the set of all natural numbers and R be the relation on N x N defined by (a,b) R (c,d), if ad (b+c) = bc(a+d), then R is

(a) symmetric only

(b) reflexive only

(c) transitive only

(d) an equivalence relation

2. A complex number z is such that $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}$. The points representing this complex number will lie on

(a) an ellipse (b) a parabola

(c) a circle (d) a straight line

3. If a_1, a_2 and a_3 be any positive real numbers, then which of the following statement is not true ?

(a) $3a_1a_2a_3 \leq a_1^3 + a_2^3 + a_3^3$

(b) $\frac{a_1}{a_2} + \frac{a_2}{a_3} + \frac{a_3}{a_1} \geq 3$

(c) $(a_1+a_2+a_3)\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right) \geq 9$

(d) $(a_1.a_2.a_3)\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right)^3 \geq 27$

4. If $|x^2-x-6| = x+2$, then the value of x are

(a) -2,2,-4 (b) -2,2,4

(c) 3,2,-2 (d) 4,4,3

5. The centres of a set of circles, each of radius 3, lie on the circle $x^2+y^2 = 25$. The locus of any point in the set is

(a) $4 \leq x^2 + y^2 \leq 64$ (b) $x^2 + y^2 \leq 25$

(c) $x^2 + y^2 \geq 25$ (d) $3 \leq x^2 + y^2 \leq 9$

6. A tower AB leans towards West making an angle α with the vertical.The angular elevation of B, the top most most point of the tower is β s observed from a point C due East of A at a distance ‘d’ from A. If

the angular elevation of B from a point D due East of C at a distance 2d from C is r, then 2 tan α can be given as

(a) 3 cot β - 2 cot γ (b) 3 cot γ - 2 cot β

(c) 3 cot β - cot γ (d) cot β - 2 cot γ

7. If α and β are roots of $x^2-ax+b = 0$ and $\alpha^n + \beta^n = V_n$, then

(a) $V_{n+1} = aV_n+bV_{n-1}$

(b) $V_{n+1} = aV_n+aV_{n-1}$

(c) $V_{n+1} = aV_n-bV_{n-1}$

(d) $V_{n+1} = aV_{n-1}-aV_n$

8. The sum of the series

$\sum_{r=0}^{n}(-1)^r\,{}^nC_r\left(\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}+\cdots m\ terms\right)$ is

(a) $\frac{2^{mn}-1}{2^{mn}(2^n-1)}$ (b) $\frac{2^{mn}-1}{2^n-1}$

(c) $\frac{2^{mn}+1}{2^n+1}$ (d) None of these

9. The angle of intersection of the circles $x^2+y^2-x+y-8 = 0$ and $X^2+y^2+2x+2y-11 = 0$

(a) $\tan^{-1}\left(\frac{19}{9}\right)$ (b) $\tan^{-1}(19)$

(c) $\tan^{-1}\left(\frac{9}{19}\right)$ (d) $\tan^{-1}(9)$

10. The vector b = 3j+4k is to be written as the sum of vector b_1 parallel to a =i+j and a vector b_2 perpendicular to a.Then b_1 is equal to

(a) $\frac{3}{2}$(i+j) (b) $\frac{2}{3}$(i+j)

(c) $\frac{1}{2}$(i+j) (d) $\frac{1}{3}$(i+j)

11. If the points (x_1,y_1),(x_2,y_2) and (x_3,y_3) are collinear,then the rank of the matrix $\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$ will always be less than

(a) 3 (b) 2 (c) 1 (d) None of these

12. The value of the determinant

$$\begin{vmatrix} 1 & \cos(\alpha-\beta) & \cos\alpha \\ \cos(\alpha-\beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$$ is

(a) $\alpha^2+\beta^2$ (b) $\alpha^2-\beta^2$

(c) 1 (d) 0

13. The number of integral values of K, for which the equation 7 cos x+5 sin x = 2K+1 has a solution, is

(a) 4 (b) 8 (c) 10 (d) 12

14. The line joining two points A(2,0),B(3,1) is rotated about A in anti-clockwise direction through an angle of 15^0. The equation of the line in the now position,is

(a) $\sqrt{3}x - y - 2\sqrt{3} = 0$ (b) $x - 3\sqrt{y} - 2 = 0$

(c) $\sqrt{3}x + y - 2\sqrt{3} = 0$ (d) $x + 3\sqrt{y} - 2 = 0$

15. The line $2x+\sqrt{6}y = 2$ is a tangent to the curve $x^2-2y^2 = 4$. The point of contact is

(a) $(4,-\sqrt{6})$ (b) $(7,-2\sqrt{6})$

(c) (2,3) (d) $(\sqrt{6},1)$

16. The number o-f integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0,0),(0,21) and (21,0) is

(a) 133 (b) 190 (c) 233 (d) 105

17. $\int (1 + x - x^{-1})\, e^{x+x^{-1}}$ dx is equal to

(a) $(x+1)\, e^{x+x^{-1}}+C$ (b) $(x-1)\, e^{x+x^{-1}}+C$

(c) $xe^{x+x^{-1}}+C$ (d) $xe^{x+x^{-1}}x+C$

18. If f(x) = x-[x], for every real number x,where [x] is the integral part of x,Then, $\int_{-1}^{1} f(x)dx$ is equal to

(a) 1 (b) 2 (c) 0 (d) $\frac{1}{2}$

19. The value of the integral $\int_{-1/2}^{1/2} \left[\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2\right]^{1/2} dx$ is

(a) $\log\left(\frac{4}{3}\right)$ (b) $4 \log\left(\frac{3}{4}\right)$

(c) $4\log\left(\frac{4}{3}\right)$ (d) $\log\left(\frac{3}{4}\right)$

20. If a tangent having slope of $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$ intersects the major and minor axes in points A abd B respectively,then the area of ΔOAB is equal to (O is the centre of the ellipse)

(a) 12 sq units (b) 48 sq units

(c) 64 sq units (d) 24 sq units

21. The locus of mid points of tangents intercepted between the axes of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$=1 will be

(a) $\frac{a^2}{x^2}+\frac{b^2}{y^2}$=1 (b) $\frac{a^2}{x^2}+\frac{b^2}{y^2}$=2

(c) $\frac{a^2}{x^2}+\frac{b^2}{y^2}$=3 (d) $\frac{a^2}{x^2}+\frac{b^2}{y^2}$=4

22. If PQ is a double ordinate of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}$=1. Such that OPQ is an equilateral triangle,O being the centre of the hyperbola, Then the eccentricity 'e' of the hyperbola satisfies

(a) $1<e<\frac{2}{\sqrt{3}}$ (b) $e=\frac{2}{\sqrt{3}}$

(c) $e=\frac{\sqrt{3}}{2}$ (d) $e>\frac{2}{\sqrt{3}}$

23. The sides AB,BC and CA of a Δ ABC have respectively 3,4 and 5 points lying on them.The number triangles that can be constructed using these points as vertices is

(a) 205 (b) 220

(c) 210 (d) None of these

24. In the expansion of $\frac{a+bx}{e^x}$,the coefficient of x^r is

(a) $\frac{a-b}{r!}$ (b) $\frac{a-br}{r!}$

(c) $(-1)^r\frac{a-br}{r!}$ (d) None of these

25. If n=(1999) !,then $\sum_{x=1}^{1999} log_n x$ is equal to

(a) 1 (b) 0

(c) $\sqrt[1999]{1999}$ (d) -1

26. P is a fixed point (a,a,a) on a line through the origin equally inclined to the axes,then any plane through P perpendicular to OP, makes intercepts on the axes, the sun of whose reciprocals is equal to

(a) a (b) $\frac{3}{2a}$

(c) $\frac{3a}{2}$ (d) None of these

27. For which of the following values of m,the area of the region bounded by the curve y=x-x^2 and the line y=mx equals $\frac{9}{2}$

(a) -4 (b) -2 (c) 2 (d) 4

28. If f:R → R be such that f(1) =3 and f'(1) = 6,Then, $\lim_{x \to o} \left\{\frac{f(1)+x}{f(1)}\right\}^{1/x}$ equals to

(a) 1 (b) e$^{1/2}$ (c) e^2 (d) e^3

29. If f(x) = $\begin{cases} (1+/\sin x/)^{a/\sin x/} , & -\frac{\pi}{6} < x < 0 \\ b , & x = 0 \\ e^{\tan 2x/\tan 3x} , & 0 < x < -\frac{\pi}{6} \end{cases}$, The n the value of a and b,if f is continuous at x=0,are respectively.

(a) $\frac{2}{3},\frac{3}{2}$ (b) $\frac{2}{3}$,e$^{2/3}$

(c) $\frac{3}{2}$,e$^{3/2}$ (d) None of these

30. The domain of the function f(x) =$\frac{1}{log_{10}(1-x)}$+$\sqrt{x+2}$ is

(a)]-3, -2,5 [∩] -2.5,-2[

(b) [-2,0 [∩] 0,1[

(c)]o,1[

(d) None of the above

31. The solution of the differential equation (1+y^2) + (x-$e^{tan^{-1}y}$) $\frac{dy}{dx}$=0, is

(a) (x-2) = K$e^{tan^{-1}y}$

(b) 2x$e^{tan^{-1}y}$ = $e^{2\,tan^{-1}y}$ + K

(c) x$e^{tan^{-1}y}$ = tan^{-1} + K

(d) x$e^{2\,tan^{-1}y}$ = $e^{tan^{-1}y}$ + K

32. If the gradient of the tangent at any point (x,y) of the curve which passes through the point $\left(1,\frac{\pi}{4}\right)$ is $\left\{\frac{y}{x}-\sin^2\left(\frac{y}{x}\right)\right\}$, then equation of the curve is

(a) y =$\cot^{-1}(\log_e x)$

(b) y =$\cot^{-1}\left(\log_e \frac{x}{e}\right)$

(c) y = x $\cot^{-1}(\log_e ex)$

(d) y =$\cot^{-1}\left(\log_e \frac{e}{x}\right)$

33. The relation R defined on set A = {x : | x | < 3, x ∈ I } by R = {(x,y) : y = | x | } is

(a) {-2,2), (-1,1),(0,0),(1,1),(2,2) }

(b) { (-2, -2),(-2,2),(-1,1),(0,0),(1,-2),(1,2),(2,-1),(2,-2) }

(c) { 90,0)(1,1),(2,2) }

(d) None of the above

34. The solution of the differential equation $\frac{dy}{dx}=\frac{yf'(x)-y^2}{f(x)}$ is

(a) f(x) = y+C (b) f(x) = y(x+C)

(c) f(x) = x+C (d) None of the above

35. If a,b and c are in AP, then determinant $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$ is

(a) 0 (b) 1

(c) x (d) 2x

36. If two events A and B. If odds against A are as 2:1 and those infavour of A ∪ B areas 3:1, then

(a) $\frac{1}{2} \leq P(B) \leq \frac{3}{4}$ (b) $\frac{5}{12} \leq P(B) \leq \frac{3}{4}$

(c) $\frac{1}{4} \leq P(B) \leq \frac{3}{4}$ (d) None of these

37. The value of $2\tan^{-1}(\operatorname{cosec}\tan^{-1}x - \tan\cot^{-1}x)$ is

(a) $\tan^{-1}x$

(b) tan x

(c) cot x

(d) $\text{cosec}^{-1} x$

38. The proposition $\sim (p \Leftrightarrow q)$ is equivalent to

(a) $(p \vee \sim q) \wedge (q \wedge \sim p)$

(b) $(p \vee \sim q) \vee (q \wedge \sim p)$

(c) $(p \wedge \sim q) \wedge (q \wedge \sim p)$

(d) None of the above

39. If truth values of P be F and q be T.Then,truth value of $\sim(\sim p \vee q)$ is

(a) T (b) F (c) Either T or F (d) Neither T not F

40. The rate of charge of the surface area of a sphere of radius r, when the radius is increasing at the rate of 2 cm/s is proportional to

(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$

(c) r (d) r^2